WATER RESOURCE OPTIMISATION IN SMALL DAMS IN THE DRY ZONE OF SRI LANKA: A TIME SERIES ANALYSIS AND STOCHASTIC SIMULATION APPROACH

by

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DECLARATION

Except where otherwise indicated in the text, this dissertation is my own work.

S. Mahendrarajah

September, 1981

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ABSTRACT

Small dams constitute the centres of traditional settlements and a predominant source of water supply in the Dry Zone of Sri Lanka. Many of the dams do not have connections with perennial supplies and hence are solely dependent upon their own catchments' rainfall for replenishment. Typically, the water resource provides for *in situ* domestic uses and for the irrigation of crops, mainly rice, particularly during the dry season.

With these two uses somewhat in conflict, a definition of community welfare is proposed. Taking the case of early maturing rice crops, it is seen that welfare can be maximised by matching the cropping calendar of the wet and dry season with the rainfall distribution. In this framework withdrawal is minimised. A major consideration in the formulation of this water resource optimisation problem involves the justification of a 'level of water application-yield' production function for rice.

Direct solution of the optimisation problem is complicated by the temporal nature of allocation and the stochastic dynamic nature of water supply in the dam. The approach adopted is to first build a simple transfer function model of water storage using rainfall and other major causal variables as inputs and allowing for the effects of uncertainty and measurement noise. Then by applying the recursive instrumental variable - approximate maximum likelihood method for identification of the appropriate model structure and identification of the parameters and their distributions therein, a statistically based time series model is made available for performing Monte Carlo simulations. Thus, the effects on storage can be observed by separately entering various supplementary irrigation policies and

rainfall years into the simulations. Further, these effects are produced in the form of probability distributions because the model parameters are sampled over their entire range of possible variation as supplied by the method of estimation.

Empirically, the approach is demonstrated using weekly data for a specific dam site in the Dry Zone of Sri Lanka. Non-linearities are introduced into the model by a modification of the rainfall, which explicitly compensates for soil moisture and evapotranspiration effects. Attention is focused on the probabilistic forecasts of the simulation model at the beginning and end of the dry season. Different cropping calendars and their associated withdrawals are examined individually in the stochastic simulation framework to determine the one that best supplements the rainfall. Commencement of the dry season's rice cultivation in the fourth week and the third week of February were observed as optimal and second best time, respectively. It is also seen that these times are different from that practised at present.

The simple stochastic modelling and simulation approach developed in this dissertation has widespread applicability. It could easily be used to investigate the optimal allocation of water resources in areas where there is a sympathetic relationship between the resource and other measurable causal variables; and it is not restricted to any particular crop. It has special advantages in developing countries as an aid in operational control since the time series representation is simple enough to be solved on a mini-computer or even hand-calculator.

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CHAPTER 1

INTRODUCTION

1.1 Dry Zone: Climate and Agriculture

Water resource-development[1] activities in Sri Lanka are concentrated mainly in the climatic region generally termed the Dry Zone. This also appears to have been the case in ancient times. For instance, Rhoads Murphey (1957, p.190) indicates:

> "...ancient irrigation works are all over the modern dry zone, and absent from the wet zone; the present wet-dry zone line coincides almost exactly with the line between the ancient irrigated and unirrigated areas".

Demarcation of the Dry Zone is primarily based on rainfall characteristics.[2] The 1900mm (75 inch) mean annual rainfall isohyet is generally accepted as a working boundary between the Wet Zone and the Dry Zone.[3] In the whole region, which covers about seventy per cent of the total land area of the Island, the greater part receives a rainfall of twelve hundred to nineteen hundred millimeters.

The rainfall is distributed in two well marked seasons, with a pronounced drought during the months from June to August. Approximately eighty per cent of the rainfall is experienced during the months from September to January. Roughly, the latter half of this period is under the sway of the North East Monsoon. In general,

1 In contrast, flood control, interbasin water transfers and related activities are of chief concern in the Wet Zone. See, for example, Cook (1951), pp.76-8; Mahawali Development Board (1977).

2 See, for instance, Farmer (1954); Thambyahpillay (1965).

3 Considered broadly, the Dry Zone then includes the ecological region of the Intermediate Zone as well. For a recent agroecologica classification, see Panabokke and Kannangara (1975).

this period is cool and wet, and also corresponds with the major agricultural season[1] that can be referred to as the *wet season*. Although short dry spells are not uncommon, rainfall during the wet season is evenly distributed and results in extensive run-off. Following this period a short dry spell occurs from February to mid-March which coincides with the harvesting activities of the wet season rice.

The second and smaller mode of the bimodal rainfall pattern occurs from mid-March to mid-May. This usually corresponds with the cropping activities of the second or *dry season*.[2] The ensuing three months are under the full dominance of the South West Monsoon. After having precipitated all its moisture in the hilly Wet Zone, the South West Monsoon crosses the Dry Zone as a dessicating wind. The drying wind accompanied by continual high temperatures imposes an aridity on the Dry Zone.

In general, temperature is stable and ranges from about 76^{o}F in December to 86^{o}F in June. Reliable uniform temperatures, with the associated solar radiation, are among the Dry Zone's greatest assets, and approach the optimum for the growth of rice (Farmer, 1954). Quite appropriately, rice production is the major agricultural pursuit of the Dry Zone, one exception being the Jaffna Peninsula.[3]

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1 Locally referred to as *Maha* and *Perumpokam* in Sinhala and Tamil respectively.

2 In the local dialects, it is known as *Yala* (Sinhala) and *Sirupokam* (Tamil); usually, a *mid season* (or *Meda* or *Idaipokam*) rice is also often referred to as a crop intensification strategy, especially under assured supply of irrigation water. The arguments of this dissertation, however, propose an early dry season rice *especially* for such situations where water supply is limited as in a small dam.

3 An intensive cultivation of subsidiary food crops and cash crops is characteristic of this region, supported by abundant, though not unlimited supplies of ground water borne by sedimentary limestone aquifers.

owever, the major problem for the intensive cultivation of rice is he shortage of water during the dry season. Water for irrigated ice production is supplied by a network of surface irrigation systems hat includes dams and lakes which are also centres of human ettlements.

.2 Small Dam Irrigation Settlements

Descriptions of life centred on small dams form part of he literary heritage of Sri Lanka. An English language classic is eonard Woolf's *The Village in the Jungle.*

> "The village consisted of ten scattered houses, mean huts made by mud plastered upon rough jungle sticks. Only one of the huts had a roof of tiles, that of the village headman Babehami; the others were covered with a thatch of cadjans, the dried leaves of the coconut-palm. Below the huts to the east of the village lay the tank, a large shallow depression in the Jungle. Where the depression was deepest the villagers had raised a long narrow bund or mound of earth, so that when the rain fell the tank served as a large pond in which to store the water. Below the bund lay the stretch of rice-fields, about thirty acres, which the villagers cultivated, if the tank filled with water, by cutting a hole in the bund, through which the water from the tank ran into the fields." (Woolf, 1913; 1974 Edition, p.6).

The complex and sophisticated maze of hydraulic works in he Dry Zone of today is considered to be a 'grafting' on to such mall-dam based systems of Medieval times. Consensus seems to exist n ascribing this system to the inhabitants of the pre-Christian era Murphey, 1957; Farmer, 1954). These early hydraulic works seem to e simple community-level attempts at water conservation. Possibly, hey could have been developed on the lines of the neighbouring ank country of South India (Murphey, 1957). Whatever the origin, uch a system seems to have brought into being in the Dry Zone a society based on a *one dam-one village* ecological pattern (Codrington, 1938: pp.1-5). Indeed, it is estimated that there are today around 3,000 such small dam settlements[1] (Panabokke, 1976). Ohrling (1977) refers to them as 'Nucleated Tank Settlements'. Very often the dam and the village in each unit share the same name; and the accepted boundary between villages seems to be the watershed (Abeyratne, 1956). Figure 1.1 is a recent mapping of the dams and lakes (or tanks and reservoirs) of the Island. The circle in the middle of the 'galaxy' of dams in the northern Dry Zone indicates the location of the dam that is the special concern of this study.

Small dams have been constructed by throwing earth in the form of a bund across seasonal streams that arise in small catchments in the undulating landscape. Capacity of dams varies, generally within 50 to 150 acre feet. Each dam is the centre of one small village settlement of about fifty families. Houses are clustered on either or both sides of the dam on relatively high ground. The highland that adjoins the housing serves as a *home-garden* to grow a mixture of annual and perennial crops. A *slash and burn*[2] system of rainfed agriculture is also practiced in the upper slopes of the landscape, mainly to grow millets and gourds. The *home-garden*, *slash and burn* system and irrigated rice together constitute an agriculture that is unique to the small dam settlements, as schematically represented in Figure 1.2.

---

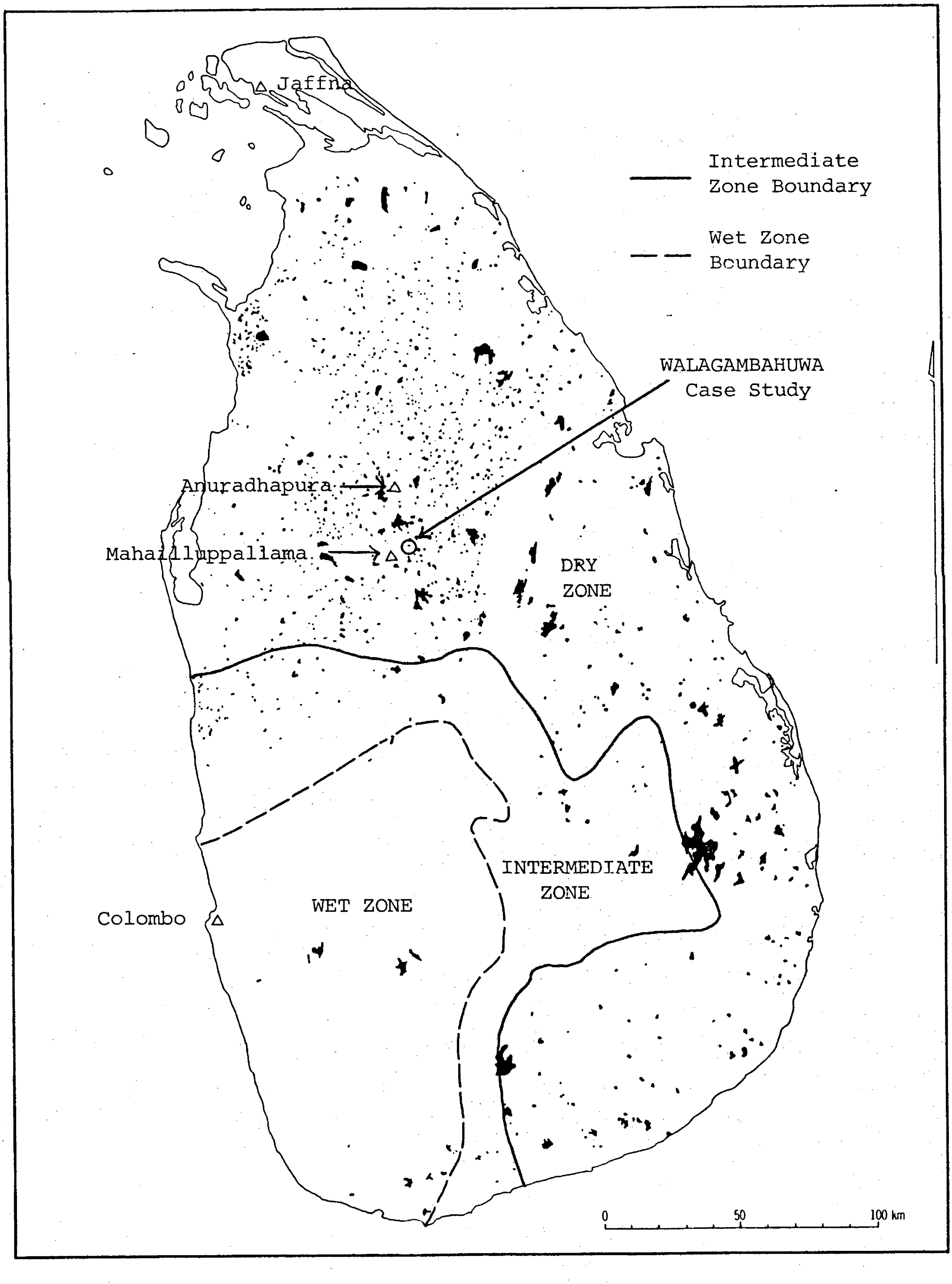
1 Agricultural statistical sources include small dams under 'Minor Irrigation Schemes' that account for 36.7 per cent (or approximately 188,000 acres) of the irrigated rice during the wet season in the Dry (and Intermediate) Zone (Department of Census and Statistics, 1977, p.144).

2 Popularly known as 'chena' (anglicized version of the local terms *hena* and *senai*).

FIGURE 1.1

DAMS, LAKES AND MAJOR CLIMATIC

BOUNDARIES OF SRI LANKA



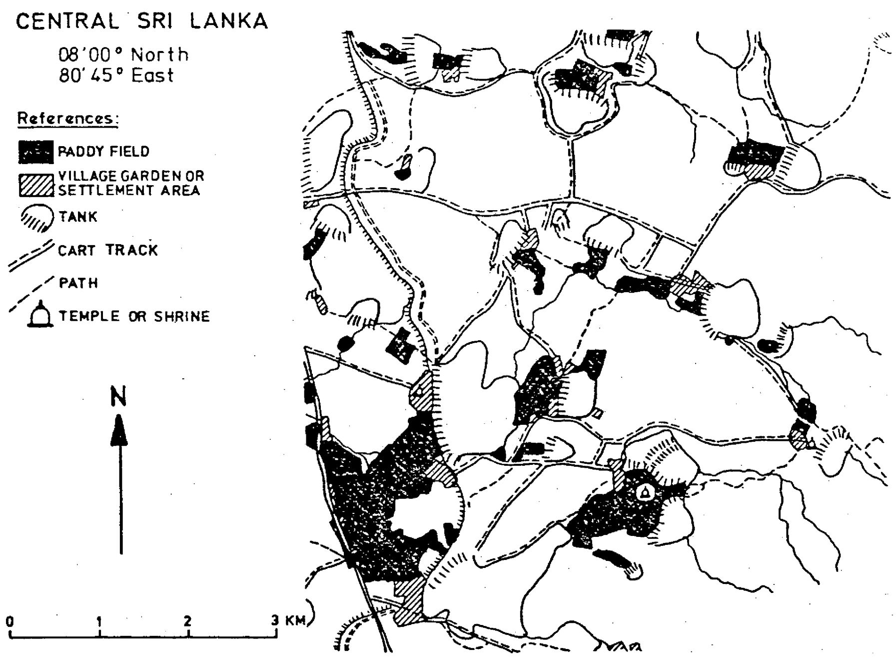
Source: Adapted from Johnson and Scrivenor (1981), p.11.

FIGURE 1.2

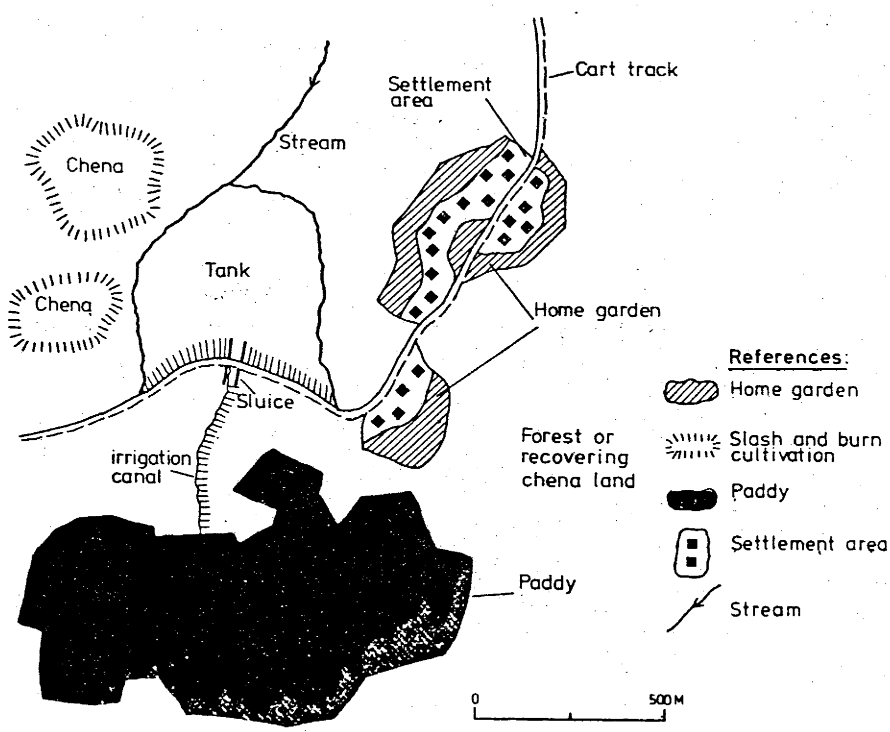
ONE DAM-ONE VILLAGE NUCLEATED SETTLEMENTS

AND AGRICULTURAL SYSTEM IN THE DRY ZONE

Settlement Pattern



Agricultural System



Source: Adapted from Ohrling (1977), pp.88-9.

The rice land of the village is a small block located just below the dam. It varies in size from thirty to fifty acres in which each family holds an inherited share.[1] The dam water is conveyed to the rice fields by way of a sluice and then by ditches or channels. Maintenance of the field channels as well as the management of the dam water is the joint responsibility of the villagers. A remarkable symbol of solidarity among the villagers is the institution of *Bethma*. This is an arrangement whereby the villagers agree in times of water shortage to cultivate only a portion of the rice land and share out the proceeds among themselves. Farmer (1957) describes this admirable system as:

> 'one under which, if the whole extent of the paddy field cannot be cultivated for lack of water, as many of the tracts as can be irrigated are divided, regardless of their ownership, between the peasants in proportion to their several holdings, and thus cultivated as a compact block with minimum waste of water' (Quoted in Leach (1961): p.170).

In addition to supporting the cultivation of rice, the dam serves as an invaluable community-asset. Its main value is as the community water supply for washing and as drinking water for the few head of cattle owned by the villagers.[2] These type of uses, the former, in particular, can be referred to as the *in situ* uses. Apart from the occasional catch of fish that the community usually shares, it is not used for any planned fish culture. Thus multiple uses of the water resource is a notable feature in small-dam settlements. Constraints to uses arise mainly because of the

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1 Greater part of the rice land is considered as '*paraveni*' or ancestral property. Social-Anthropological Studies of Leach (1961) and Obeyesekere (1970) discuss in considerable detail the rice land tenure and inheritance.

2 Village drinking water is typically drawn from wells sufficiently near the dam for their water level to be considered synonymous with that of the dam itself.

variability in the amount of water that becomes available in the dam through the year.

1.3 Water Storage-Depletion in Small Dams

The majority of small dams are isolated and not reached by the modern hydraulic network. They are completely dependent upon their own catchments for replenishment. It is these dams with which this dissertation is concerned.

In such dams, water storage at a particular point in time is highly variable. However, the storage through the year generally exhibits a bimodal pattern very much similar to that of the rainfall. Water level builds up during the wet season and reaches its highest magnitude by the end of December. During the subsequent months it gradually recedes until April. From April until the end of May, there is a significant addition to storage levels caused by the dry season rainfall. This second peak is followed by an accelerated pace of depletion evidently as a consequence of the prolonged drought from June to August. The lowest level of water storage is approximately 10-20 acre feet by the end of August. However, there are years when the dams become completely dry.

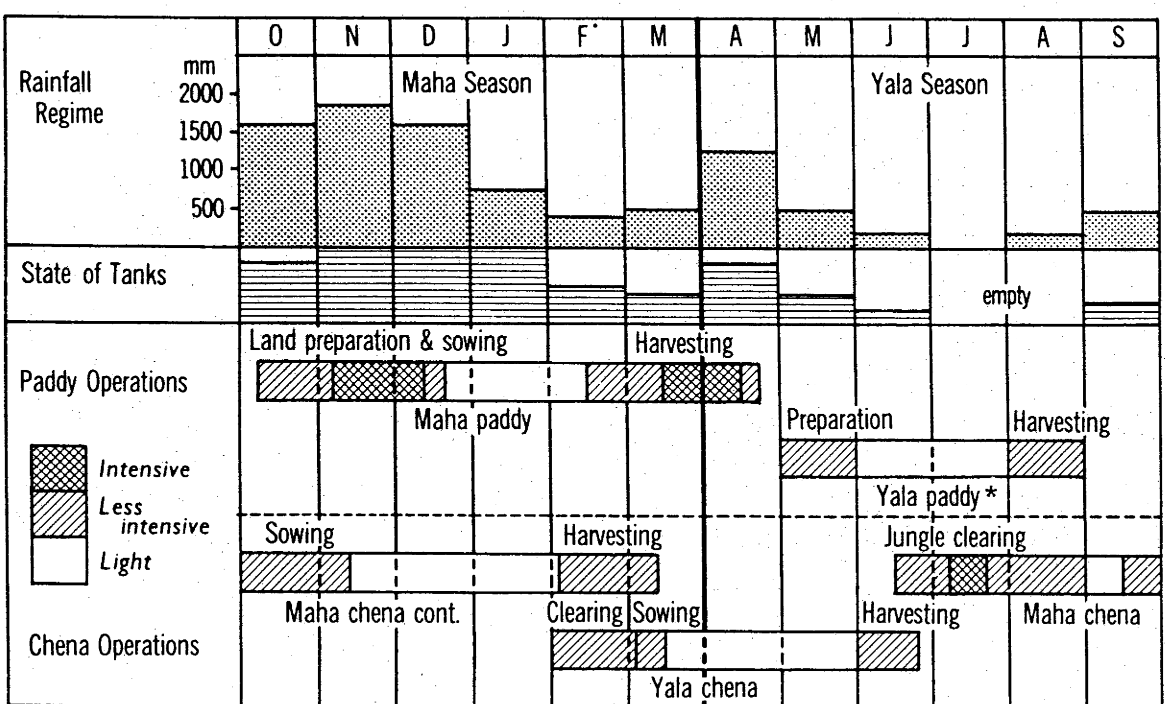
The above natural storage-depletion behaviour seems to have received explicit consideration in the established irrigation practices of the villages.

Traditionally, in many dams, irrigated rice has been cultivated in only one season with the land preparation activities commencing in December. In effect, it has been a delayed wet season rice crop. The wet season's rainfall appears to have been used to advantage only for land preparation. This delay seems to have been motivated by a custom of waiting to ascertain the water storage. Judgements regarding the extent of cultivation appear to have been largely based on the water level in December. The delayed planting of rice also gave time for the 'rainfed' *slash and burn* or chena cultivation which necessarily commenced with the onset of the rains in the wet season.

Dry season rice cultivation in small dams under the traditional practice has been a gamble if not a dream. The key elements of the rainfall pattern, dam storage and the cropping pattern are represented schematically in Figure 1.3.

FIGURE 1.3

A SCHEMATIC REPRESENTATION OF RAINFALL DISTRIBUTION, DAM WATER LEVELS AND TRADITIONAL CROPPING PATTERN IN THE DRY ZONE



* Occasionally

Source: Adapted from Johnson and Scrivenor (1981), p.42.

1.4 The New Approach to the Management of the Water Resource

Sustaining production levels in *slash and burn* or chena cultivation with increasingly shorter rotation and poor fertility maintenance is of concern and practical approaches to attain increases in productivity of the rice land are being sought as a way of raising incomes. Particular emphasis is being placed on the need to raise cropping intensity. This involves early sowing of the wet season rice and the accommodation in the cropping calendar of a dry season cultivation. The aim essentially is to raise the wet season rice crop without irrigation as far as possible, and thereby conserve dam storage for irrigation of the dry season crop. This constitutes the most recent approach to water resource management of small dams.[1]

The practicability of this management approach has already been demonstrated for a few dams in the Anuradhapura district. Moreover, the farmers using these dams have continued to follow the new practice. Adoption of the practice in adjoining villages has also been observed providing some evidence for the acceptability of the approach. The economics of such new management have been documented by, for example, Mahendrarajah (1978) and Sivapatham (1979). Though useful, these studies fail to give adequate consideration to the crucial role of the water resource. In fact, the latter study is confined to the analysis of resource inputs other than water, although it implicitly recognizes the vital role and value of the water resource in the dry season.

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1 This formed the main theme of the Cropping Systems Workshop of the Department of Agriculture in 1976. See *the Proc. Cropping Systems Workshop*, Maha Illuppallama, Sri Lanka, April 20-21, 1976.

Thus the problem of water resource management in small dams arises in the dry season and concerns efficient use of the natural storage that has built up by the end of the wet season. However, the efficient allocation of dam water for the dry season crop is not straightforward for two reasons:

(i) the temporal or dynamic behaviour of the storage levels (dependent on factors like the changing rainfall and weather) and the temporal nature of water allocation; and

(ii) the existence of multiple uses for the water resource of the dam.

Therefore this management problem is chosen as a worthwhile and appropriate topic for investigation.

1.5 Aims and Scope of the Dissertation

The research in this dissertation concerns the effective temporal allocation of the water supply to the irrigation of a single crop, namely rice. However, the approach extends so that guidelines can be provided for optimal irrigation decisions with respect to other possible dry season crops in the rice land.

The main focus is upon the welfare of the small dam community when it has multiple goals such as rice production and *in situ* uses. An important component of the work consists of the discussion and derivation of the optimal allocation mechanism. Our solutions require consideration of the changing nature of the water supply in the dam. The development of an appropriate analytical procedure constitutes a valuable component of the study.

Efficiency in the use of scarce and often inaccurate data is a prime consideration. The procedure developed requires only a modest amount of data and allows for measurement of errors on them. It also appears to have long-term value as an aid for irrigation decisions and potential accessibility on a mini computer. Finally, this dissertation undertakes empirical application of the procedure for water resource optimisation using data for a selected small dam.[1]

The specific objectives of the study are spelt out at the end of the formulation of the optimisation problem in Chapter 2. The broader aims of this research can be stated as follows:

(i) to develop a conceptual model for water resource optimisation in small dams;

(ii) to develop an analytical model to assess allocation decisions; and finally

(iii) to empirically demonstrate the approach in a real small dam situation.

1.6 Organization of the Dissertation

This dissertation is presented in six chapters. Chapter 1 has introduced the topic, raising the issue of allocation of water for the dry season. It has also stated the broader aims and scope of the research.

Chapter 2 undertakes the discussion and formulation of the optimisation problem. The nature of the issue of maximising community welfare under conflicting goals of water use is outlined.

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1 The characteristics of the small dam and the data sources are considered in Appendix A.

This chapter also deals with the temporal demand for irrigation water for rice, with the more fundamental choice of treating water as a variable input in the production process. The review and adoption of a 'level of water application - crop yield' production function leads to the definition of the optimisation as a constrained linear programming problem. Methodological pre-requisites for solution are dealt with in Chapter 3. Emphasis is placed on a system analysis approach to parameterise water storage as a 'linear dynamic system'. For parameter estimation, useful techniques of recursive time series analysis are also reviewed. Estimates and statistics emerging from the estimation provide the input for the development of a stochastic simulation model for storage.

In Chapter 4, by explicitly compensating for hydrologic elements that contribute to 'nonlinearity', an effective rainfall-storage system is obtained which simplifies the methodology. Results of the empirical application are also presented in this chapter, highlighting the sensitivity of storage at the beginning and at the end of the dry season. Chapter 5 is devoted to the formulation of an irrigation policy, definition of strategies and their evaluation using the framework provided in Chapter 4. Results also include findings that relate to different rainfall years. Chapter 6 provides a summary and a concluding note on the value and limitations of this approach, and suggests future extensions to this research.

CHAPTER 2

THE OPTIMISATION PROBLEM

The issue of optimisation of water use in small community-dams is discussed in this chapter and it constitutes the basis for the identification and definition of the specific objectives of this dissertation. The text is divided into five sections. Sections 2.1 and 2.2 identify the requirements for an explicit welfare consideration. The physiological rice-water relationships constitute the foundation for the technical water - crop yield response. These are dealt with in Sections 2.3 and 2.4 and provide the essential preliminaries for the identification of the appropriate optimisation model presented in Section 2.5. In the light of the requirements of the latter, specific objectives are set out in Section 2.6 to obtain solutions for a selected dam.

2.1 Optimisation: A Definition

Economics is concerned with rational choices. Optimising is finding a 'best' choice among possible alternative choices in relation to a set goal or goals. A formal 'optimisation model' consists of a 'choice space' which is the set of comparable alternatives, an 'objective function' which describes how alternatives are to be compared and a 'feasible region' which is a subset of the choice space and contains those alternatives that are eligible for choice (Day, 1977). As the objective function or goal is central to an optimisation issue, the identification and description of goals is the logical first step in the formulation of strategies for the management of water resources. Furthermore, in the process, a set of standards emerges, with which it is necessary to evaluate the relative merits of alternative strategies.

The ultimate goal in the management of natural resources is well recognized. A consensus seems to exist that the resources be utilized efficiently to make maximum contribution to the community welfare. But the concept of community welfare is truly nebulous. The insusceptibility of welfare to quantification renders it impotent as an operational criterion for economic analysis. Therefore, criteria involving measurable magnitudes must be substituted for such an ultimate but non-operational goal.

2.2 Welfare Criteria for Small Dam Water Resources

The choice of criteria is conditioned by the nature of the resource under consideration. Also, the search for appropriate welfare criteria for water resources will be facilitated by an understanding of the welfare concept and this is treated first here.

2.2.1 The Concept of Community Welfare

The concept of community or social welfare is usually related to the satisfaction or well-being of individuals who make up that community. The general postulates are:

(i) that the social welfare depends solely on the utility level of each person in the community; and

(ii) that the individual alone is the best judge of his utility or satisfaction (Mishan, 1960).

So at the individual level, the goal of optimising the water resource could be seen as making one 'better off' in one's own estimation (Mishan, 1969). In practice, however, this is difficult to establish. In the operational context, there is a need for an index of choice expansion (Mishan, 1969).

The components of such an index of choice expansion may include specific economic objectives to which an individual might aspire. The scope of such an index will very much depend upon how complete its specification is. However, there is always a hierarchy of goals in which the goal of one action becomes the means to achieve further goals. In so far as the attainment of a reasonable level of income is a means of accomplishing other goals, then the level of income could be treated as an index of a person's welfare. However, the transition from individual to community income requires further assumptions.

One possible operational criterion or objective for the community is the income via production. However, accepting this as the index of welfare implies that a desirable distribution of income is also achieved. Fortunately, the equity in distribution of rice lands is remarkable within the community in small dam based villages.[1] The production of the largest 'social pie' in terms of production with the available water could, therefore, be adopted as a welfare goal.

However, the production criterion alone is inadequate for small dams, as they provide alternative benefits via conservation as well.

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1 This feature is attributed to the inheritance pattern and the value system of these communities (Obeyasekere, 1970).

2.2.2 The Water 'Good'

The water in the small dams is unique in that the type of use to which it is put determines its nature as a resource. It has two dimensions as a 'good'. First, the water resource exists as a *private* good. Water utilization in crop production is such that the initial use reduces the total supply available for subsequent uses by the entire amount of the original use. On the other hand, the water resource, when used for other *in situ* community purposes, has the characteristics of a *public* good, since the initial use does not result in the reduction of supply (Davis and Winston, 1967). However, it must be noted that in either use the water is free to the community and contributes to its welfare.

An obvious conflict, nevertheless, does exist among the two welfare goals. The increment to welfare via the use of water as a private good leads to a reduction of storage. The reduction in the level of water in the dam in turn is likely to affect the welfare through its subsequent constraints on *in situ* uses. On the other hand, a preoccupation with conservation benefits precludes high benefits from crop income. Clearly, there is a need for a trade-off between these two welfare goals.

One final consideration of the small dam water resource as a private good should be mentioned. That is, the resource *may be expected* to have the so called common-pool problem[1] which arises due to the 'fugitive' nature of the supply: the water is no-one's property until it is appropriated and actually used. Since the

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1 For a discussion of such a problem with respect to common-property water resources, see Hirshleifer et al (1977, pp.61-3).

rights to the dam-water can only be obtained by actual use, the users are induced to withdraw at a rate greater than would otherwise be rational. This is very likely due to the fear that the withdrawals of others will lower the water level in the dam. This would mean a divergence of private and community costs of use and implies that private decision making is not desirable in dam-water management. In actual fact, the community that depends on the dam avoids this problem by making withdrawals of water subject to a decision of the Cultivation Committee,[1] a democratically elected community body.

2.2.3 Operational Welfare Criteria for Conservation

The benefits accruable via crop production and income are obtained only at the end of a period of water use. On the other hand, the benefits from *in situ* uses or conservation are derived day-to-day in a continuous fashion. In order to bring the conservation benefits into parity with crop income, they have to be valued in comparable metrics and be aggregated over a period of time which is the crop duration. These aspects add to the difficulty of choice of appropriate operational welfare criteria for conservation.

Available methods of valuation of non-commensurables are discussed, for example, by Thampapillai (1976) and Sinden and Worrell (1979). Generally, these methods value the conservation benefits in monetary terms using, for instance, a 'willingness-to-pay' criterion. But such valuations are cumbersome and unrealistic in the present context. Two aspects of the *in situ* uses are important

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1 The Paddy Lands Act 1958 provided for the Cultivation Committee which is vested with the power to make decisions regarding the management of water.

to note in this regard:

(i) the vital nature of the uses, in contrast to a recreational service which is amenable for a straightforward pricing; and,

(ii) no water market exists in general for water resources and for associated services, including recreation, in Sri Lanka.

Non-monetary criteria are therefore favoured for measuring conservation benefits of small dam water resources.

An obvious index for welfare is the level of water storage itself. Water storage below a certain level is likely to affect the *in situ* uses to which it is put. In other words, it is reasonable to assume that the conservation benefits are an increasing function of the level of storage up to a ceiling (or a minimum storage requirement). Beyond such a ceiling the community will be indifferent to the level of storage as far as the conservation benefits are concerned. The minimum storage requirement is the revealed preference of the community and could be obtained by inquiry.

The probability of the storage receding below the 'minimum' community requirement is strongest towards the end of the agricultural year. This is due to the rainfall pattern and the unique storage behaviour of the dams. So the 'end of the year' storage can be used as a surrogate for the overall conservation benefits to the community. An irrigation (or withdrawal) strategy for crop production is not likely to affect the conservation welfare so long as it does not drive the storage below the 'minimum' level. Furthermore, below the 'minimum', a strategy that leaves behind a higher amount of storage at the end of the year can be treated as superior to one that results in lower storage.

From the foregoing it could be surmised that crop-income and 'year-end storage' level provide two operational welfare criteria for the optimisation of water resources of small dams. With this established, a discussion of the role of water in rice production is in order.

2.3 The Rice-Water Relationship

The demand for water for crop production in the community is not a direct but a *derived* demand. It is also temporal in nature and naturally has a vital biological and physiological basis (see, Kramer, 1969), which obviously is crop specific.

The crop under consideration in this dissertation is an early maturing variety of rice.[1] As discussed earlier, the issue of allocation of water concerns the cultivation of rice during the *dry season* and encompasses the problem of distribution within its growing season. Since efficient allocation of water requires a knowledge about the growth of the rice plant and water requirements, the rest of this section is devoted to briefly aspects.

2.3.1 Growth Stages and Yield Components of the Rice Plant

In the growth cycle of the rice plant three distinct phases may be discerned, namely:

(i) the Vegetative Phase, which extends from the germination of seed to the initiation of the panicle (or PI);

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1 A rice variety maturing over a period of 3-3½ months is known as an 'early maturing rice' and the number of months define the 'age class' of a variety.

(ii) the Reproductive Phase, extending from PI to flowering; and

(iii) the Ripening Phase which extends from flowering to full maturity of grains.

Normally, the duration of both the reproductive and ripening phases are more or less constant regardless of the age class to which the variety belongs. In contrast, the vegetative phase is variable among varieties; shorter in early maturing varieties and longer in late maturing ones.

In the vegetative phase, the rice plant gains the vegetative mass and tillers[1] required before it will initiate the panicle primodia. The panicle initiation (PI) marks the commencement of the reproductive phase. However, the PI may begin before the maximum tiller number is reached. The reproductive phase ends with the flowering which also marks the commencement of the ripening phase. The latter covers all the stages through which the grains develop. Critical among the stages are the *milky* stage, where the content of the grains is milky in consistency, and the *dough* stage, just before the maturity of the grains. However, as a whole, all these three growth phases, whose durations are shown in Table 2.1, have important bearings on grain production.

The grain yield of the rice plant is a function of three components:

(i) the number of panicles per plant;

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1 Refers to 'branches' of rice plant (or cereals) and each branch or tiller can potentially bear a panicle.

TABLE 2.1

APPROXIMATE DURATIONS OF GROWTH PHASES OF THE RICE PLANT

Phases	Duration
Vegetative Phase	25 to 65 days and specific to a variety 25-35 days for an early maturing variety
Reproductive Phase	Around 35 days regardless of variety
Ripening Phase	25-35 days regardless of variety

Source: Adapted from UPLB and IRRI (1970), p.26.

(ii) the number of filled spikelets per panicle; and

(iii) the mean weight of individual grains.

The number of panicles is a function of the number of productive tillers. It is to a large extent determined during the vegetative phase. The number of spikelets per panicle depends upon the activity of the plant during the reproductive phase while the grain weight is determined during the ripening phase. Unfavourable growth conditions during the reproductive and ripening phases can, therefore, result in small panicles with a high proportion of unfilled grains (or chaff). On the other hand, the number of panicles is a function of the number of productive tillers. Poor management during the vegetative phase results in fewer productive tillers.

2.3.2 <u>Water Duty for Rice</u>

The water requirement for normal growth of a crop is known as the consumptive use. Of the consumptive use, around 99 per cent is utilized by the crop to satisfy the needs of evapotranspiration. The evapotranspiration is a vital process and also keeps the plant cool. It is not only dependent on the crop but is also a function of other environmental and soil factors. In the Dry Zone of Sri Lanka, the evapotranspiration demand of rice during the dry season is approximately 6-8mm/day (Alles, 1967). This works out to a total of 600-800mm for a rice crop of 100 days duration.

The total infield water requirement has to include not only the consumptive use but also allowances for other losses such as seepage and percolation. Seepage and percolation refer to losses into the soil via lateral flows and vertical flows respectively. In addition to such losses, wet land cultivation of rice requires substantial amounts of water for land preparation. The total amount of water required per unit area for the cultivation of a rice crop is known as the *water duty* of rice.

The water duty is obtained from the irrigation and the rainfall that the crop receives. More specifically, the *on-field* water duty is expressed as a ratio of the quantity of water received to the area of the crop. The unit used for water duty in Sri Lanka is acre-inch or acre-foot. Sometimes an *ex-sluice* water duty is also used in irrigation systems in order to incorporate the conveyance losses. However, such a concept is not very useful in the context of small dam rice lands. The rice lands in small dams are not only relatively small in extent but also adjacent to the dam. In the present context then, the term water duty refers to the *on-field* water duty.

The estimates of water duty of rice in the second season range from 4.1 to 6.3 acre-feet as shown in Table 2.2. Allowing

TABLE 2.2

ON-FIELD WATER DUTIES OF RICE AND IRRIGATION REQUIREMENTS IN AN AVERAGE RAINFALL YEAR IN THE DRY ZONE OF SRI LANKA

(in acre-feet per acre)

Source	On-field Water Duty		Rainfall		Irrigation Requirement	
	Wet Season	Dry Season	Wet Season	Dry Season	Wet Season	Dry Season
Murakami and Vignarajah (1967)	4.9	6.3	3.1	0.9	1.8	5.4
Alles (1967)	3.6	4.1	3.1	0.9	0.5	3.2

Note: Also quoted by Chambers (1978, p.28).

for the rainfall during the season, the requirement via irrigation is estimated at between 3.2 and 5.4 acre-feet. These estimates are derived from large irrigation systems. Experimental estimates of water duty for rice crops below small dams are not available. The author's experience of a small dam dry season cultivation indicates a very much lower water duty figure of the order of 1½ to 2 acre-feet. In one particularly stringent management situation, around 30 acres of rice land required a total irrigation of 8-10 acre-feet.

Two reasons can be advanced for the much lower water duty for rice under small irrigation dams.

(i) a hydrological aspect peculiar to small dams: there seems to exist a continuous seepage influence from the dams which are generally nearby. Such an influence is likely to keep the ground water level relatively close to the surface in the rice land.

(ii) the stringent water management itself: the strategy of irrigation of dry season rice in small dams does not aim to provide water sufficient for maximum and normal growth. Rather, it supplements the incident rainfall with an amount of irrigation just enough for moderate rice yields.

2.3.3 Importance of Water in the Growth Phases

The provision of a lower 'water duty' to the rice crop is likely to affect the growth and yield performance of rice. These reauctions in yield could be manifested in one or more of the yield coponents. When the water 'level' provided in the soil is indequate for the plant to freely meet the evaporative demand, the plnt is said to be under 'moisture stress'. As yet, the specific efects of moisture stress on the physiology of the rice plant are no well established. Nevertheless, Kramer (1969), for example, dos discuss the effects on many of the vital aspects related to grwth. Such a broad understanding is sufficient for our purposes.

In the rice crop, moisture stress during the early veetative phase appears to have adverse effects on plant-tillering wih consequent reduction in grain yield (UPLB and IRRI, 1970). Neertheless, it has been observed that if water is made available afer a prolonged drought in this growth phase, the crop tends to make up its growth deficiency particularly with additional fertilizer application. However, the maturity of the crop is likely to be delayed. On the other hand, if the water stress is experienced in the reproductive and ripening phases, a further reduction in yield takes place because of its effects on the other yield components. Specifically, it leads to a reduction in the grains per panicle, percentage of filled grains and the mean weight of grains (De Datta et al, 1973).

The effects of water stress are believed to be more pronounced at certain 'critical' growth stages than at other stages. Such critical stages also seem to coincide with the periods in which the plant uses most water. Salter and Goode (1967) conclude that cereal crops show a marked sensitivity to water stress during the formation of reproductive organs and during flowering. Matsushima (1962) reported that the rice plants are more sensitive to water stress from 20 days before heading to 10 days after heading.[1] Rice crop susceptibility to water shortage during the reproductive and ripening phases has been reported by Hiler et al (1971) to be around two times that during the vegetative phase. However, there is evidence that for many high yielding varieties of rice no growth stage is more critical to moisture stress than others (De Dutta et al, 1973). The same study also concludes that the susceptibility is dependent on the age class of the variety.

In early maturing varieties of rice, it seems difficult to single out one or two stages of growth as 'critical' to moisture

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1 Denotes the emergence of the panicle tip out of the flag leaf-sheath. The heading stage is approximately 10 days before flowering.

stress. For example, De Dutta et al (1973) conclude that in these varieties all the growth stages are equally susceptible to moisture stress; they have observed that moisture stress affects yield equally adversely even when it occurs during the vegetative phase. This could be attributed to the fact that the vegetative phase in these varieties, unlike other age classes, is short and it is more 'valuable' to build up the essential vegetative mass.

An important observation that emerges from the above discussions is that the soil moisture level must be adequately maintained during the *whole* crop duration, particularly for early maturing rice varieties which are of importance in the present study. This has implications for water application. Since, the evapotranspiration in the field is relatively constant, the logical approach is to distribute the 'water duty' in equal amounts in a number of applications through the crop duration. This strategy is *especially* relevant when the supply is not adequate enough to provide the full 'water duty' requirements.

2.3.4 Irrigation Scheduling

The non-rainfall component of the 'water duty' is applied in the form of irrigation water. The scheme of applying the irrigation water to the crop is known as 'Irrigation Scheduling'. The irrigation schedule sets out the amount and time of irrigation throughout the cropping duration.

Historically, the earliest scheduling was purely based on farmers' judgement and experience. It took into consideration the appearance of the crop and it proved to be wasteful. Improved scheduling techniques are based on a climatological approach as discussed, for example, by Pruit and Jensen (1955), and Van Bavel and Wilson (1952). They make use of soil and climatological information. But they also utilize many measuring devices.

A judicious supplementation of the rainfall for rice from water in small dams needs to be based on a climatological approach with the primary variable of interest being the record of rainfall. The scheduling could also be aided by such visual observations of the soil moisture status as the presence of standing water, or the dryness or cracking of the soil. Although it has already been decided here that the 'water duty' or demand should be at a constant level within a season, the random nature of the primary supply, rainfall, means that the supplementary supply, irrigation, has to be provided in variable doses.

Clearly then, the issue of optimum use of the water resource must address some form of optimum scheduling. This also must take into account the productivity of water. Before turning to the optimum management of dam-water, therefore, further discussion is needed on the empirical work of economists in considering water as an input in rice production.

2.4 Water Input in the Production Process: A Review

In the treatment of water as an input to agricultural production, economists have adhered to one of two schools of thought, as discussed, for example, by Flinn (1968). One group seems to be dominated by the idea that water is a vital necessity to crops and that each crop has a unique water requirement. It also believes that water is an input which is complementary to other physical inputs in the production process. On the other hand, the second group contend that water like any other input has substitution possibilities and has a production response function.

These two strands of 'thought' have greatly influenced the literature in irrigation economics and its concepts. An explicit distinction is crucial in the evolution of any management strategy. In particular, it will be seen that the concept of 'variable input' is more relevant for small dam situations.

2.4.1 Water: Complementary Input

The concept of 'unique water requirement' has received frequent attention and dominates the literature of irrigation economics. Very often it is a basic assumption in irrigation development planning and water resource investments. Steiner (1964) assumes unique water requirements for crops in water resource investment evaluations.

In the economics of irrigation of agricultural production, it has been a convenient concept. Clark (1970) reviewed the economics of irrigation of alternative crops with 'unique water requirement' estimates. However, this concept becomes sterile when it confronts the economics of irrigation of a single crop. Generally in the irrigation of a single crop, the 'economics' of irrigation usually means little more than the attainment of greater efficiency in water use by the reduction of waste. The popular proposition is that more sparing and less wasteful use of available water resources enables the community to increase the extent of cultivation. Following this line of reasoning, the benefits accruable to economic (or efficient) use of water is derived in terms of either additional land brought under cultivation or an increase in the cropping intensity of the land already in cultivation, or both. Chambers (1976; 1978) adopted this kind of reasoning for optimal management of water resources for rice production in the Dry Zone of Sri Lanka.

The complementary role of water becomes evident in the discussions of a rainfed production system. Additional benefits accruing to the application of irrigation water to hitherto purely rainfed systems is very often viewed in terms of increases of marginal value products (MVPs) or productivity of the other inputs. Such a postulate implies an upward shift in the production function on the provision of irrigation. Notably water is not a variable input in such a production function. Yet, this constitutes the core for many studies which attempt to assess the impact of irrigation. The framework for such an analysis is either a 'cost-benefit' approach or a production function. A production function is adopted by Desai (1973) and Sadeghi (1978). The same approach has also been used, for example, by Levine (1966) to highlight the higher impact of irrigation in the dry season in comparison to that in the wet season in a selected region. However, a fundamental point to bear in mind is that these analyses are not concerned about the supply of the water resource. At best it is assumed as 'free and plenty'.

2.4.2 Water Response Functions

The alternative theory treats water as a variable input in the production process. It emerges from the proposition that crops exhibit differential responses to variation in the quantity of water made available. This provides the necessary physical basis for analysis within the conventional production function framework.

Empirical studies to estimate the productivity of water simply use planned experiments to vary the quantity of water applied to a crop. At the farm level, there have been many attempts to make such estimates. Two of the noteworthy examples are Hopper (1965) and Naik (1965). These studies have made use of quantity measures of water at the farm 'headgate' of sample farms to specify whole-farm production functions. Such a technique is not relevant in the context of a small dam which concerns only a *single* 'community farm'. Furthermore, the objective in such dams is not simply to maximise productivity of water in rice production but to ensure the sustenance of a crop of rice.

For the small dam situation, a relevant set of concepts seems to exist in moisture stress-yield functions.

2.4.3 Moisture Stress - Yield Functions

There has been a distinct group of studies which infers the importance of water to crops indirectly. Instead of quantity measures of water, they incorporate a drought index or an index of stress reduction in the specification of production functions.

The relevant agricultural engineering literature is substantial. A few seminal papers on the above are noteworthy. Knetsch (1959) made use of drought days to define critical levels of soil moisture deficit in a production function of corn yield. Similar functions for rice are given by Wickham (1973) and Bhuiyan and Sumayao (1978). The obvious value of irrigation in increasing yield via the reduction of drought days was highlighted by Parks and Knetsch (1960) and Reutlinger and Seagraves (1962). These studies incorporated into production functions, the number of days of stress reduction via irrigation. Recently, in a study to examine the differential performance of rice within an irrigation system, Asnawi (1981) has considered stress days and field water depths in production functions. A greater insight into such functions has been provided by Beringer (1961). Beringer (1961) drew heavily from the more fundamental soil-plant-water relationships. He also postulated the operation of the *law of diminishing marginal returns* between the decreasing soil moisture stress and crop yield.

Nevertheless, the above studies do not give explicit recognition to the temporal nature of water use. Moore (1961) was, perhaps, the first to identify the problem of allocation of water over time. However, Yaron's (1971) contribution is more valuable in the course of water resource optimisation research. Yaron (1971) distinguished between two types of water-yield relationships; namely:

(i) the yield with the total water input having a fixed intraseasonal distribution; and

(ii) the yield with flexible and dated water input.

The studies of Stewart and Hagan (1969), Hagan and Stewart (1972) and Ellis (1972) adopt the first approach which involves estimation of the relationship between water shortage and yield by regression techniques. The difficulty of estimation of the second type of function was recognised by Yaron (1971). He advocated the derivation of 'growth rules' by simulation in a dynamic programming framework, with growth stages and states a transition function to update the yield from period to period. In fact this approach has been predominant in recent water-resource optimisation studies.

2.4.4 Optimal Dam-Water Management

Optimal water allocation issues usually recognize the temporal nature of the water demand. Most of the studies have employed a dynamic programming framework for analysis which allows sequential decision making. Normally the crop growth duration is divided into a number of stages at which irrigations are given. The decision on irrigation quantities at each stage is made considering the state of the water storage and crop growth towards attaining maximum profit. The optimisation studies[1] could be classified, depending on the nature of water supply and demand at each stage, into three groups.

1. Water supply stochastic and demand non-stochastic: Anderson (1968) and Butcher (1971) analyse the optimisation within such a system.
2. Non-stochastic supply and stochastic demand: Burt and Stauber (1971) assume a given supply of water and a variable demand depending on rainfall, in optimal allocation of water for corn in a sub-humid climate.
3. Stochastic supply and stochastic demand: This class of models has been associated particularly with Norman Dudley; for example, Dudley (1970), Dudley et al (1971a; 1971b) and

1 This review is based on Day and Sparling (1977).

Dudley and Burt (1973) consider such a model. A stochastic dynamic programming framework has been employed to determine the optimal temporal water allocation, intra-season irrigated extents and pre-season planning of acreage to plant.

However, none of the studies in the above groups consider a dynamic water supply that exhibits a marked temporal behaviour as in the small dams in Sri Lanka. Besides, as discussed earlier, the intraseasonal demand of water for early maturing rice is constant. Further, it will be seen that the present study is centred only around a single level of production associated with a 'minimal supplementary irrigation policy'. Clearly then, a dynamic programming framework with the objective function defined to maximize production (or profit) is not relevant. Before turning to the appropriate approach, clarification is necessary on the meaning of the 'minimal supplementary irrigation policy'.

2.4.5 Minimal Supplementary Irrigation Policy

For optimal allocation of the water resources in small dams, water input is best treated within a production response function framework. And within the crop growth duration, an equal distribution of water has been shown to be favourable since the water duty (or requirement) can be considered to be the same at (or constant over) all stages. Moreover, the existence of a yield response function to soil moisture stress reduction via irrigation has also been established. Thus the fixed intraseasonal distribution and the response function together define an underlying simple neoclassical production function for rice yield in response to the *level* of water (irrigation plus rainfall) application. Sophisticated considerations of the intraseasonal distribution of application seem unnecessary. It is, therefore, reasonable to assume a response function between yield of rice and fixed intraseasonal *level* of water (rainfall plus irrigation) application as shown in Figure 2.1.

With all the other factors of production remaining unchanged, the provision of water for normal growth (i.e., the 'water duty'), x_{max} yields maximum grain output, y_{max}. At the other extreme, there exists a level of application below which it results in complete crop loss. In other words, only a level of water application above x_{min} in Figure 2.1 will ensure a crop of rice. The yield target in small dams could be visualized as one lying between these two extremes and we can call it y^* which corresponds to a level of application x^* such that $x_{min} \leq x^* < x_{max}$.

During the crop growth, water needs to be applied a number of times. Of each application[1], x^*, a significant component can be met by rainfall. Only the balance needs to be provided by supplementary irrigation. A judicious supplementary irrigation policy which ensures a crop taking into account the incident rainfall can be defined for our purposes as a 'minimal supplementation policy'. Such a policy has been observed to result in an average yield of 2 tons per hectare which is comparable to the wet season yield in the dam under a purely 'rainfed' situation.

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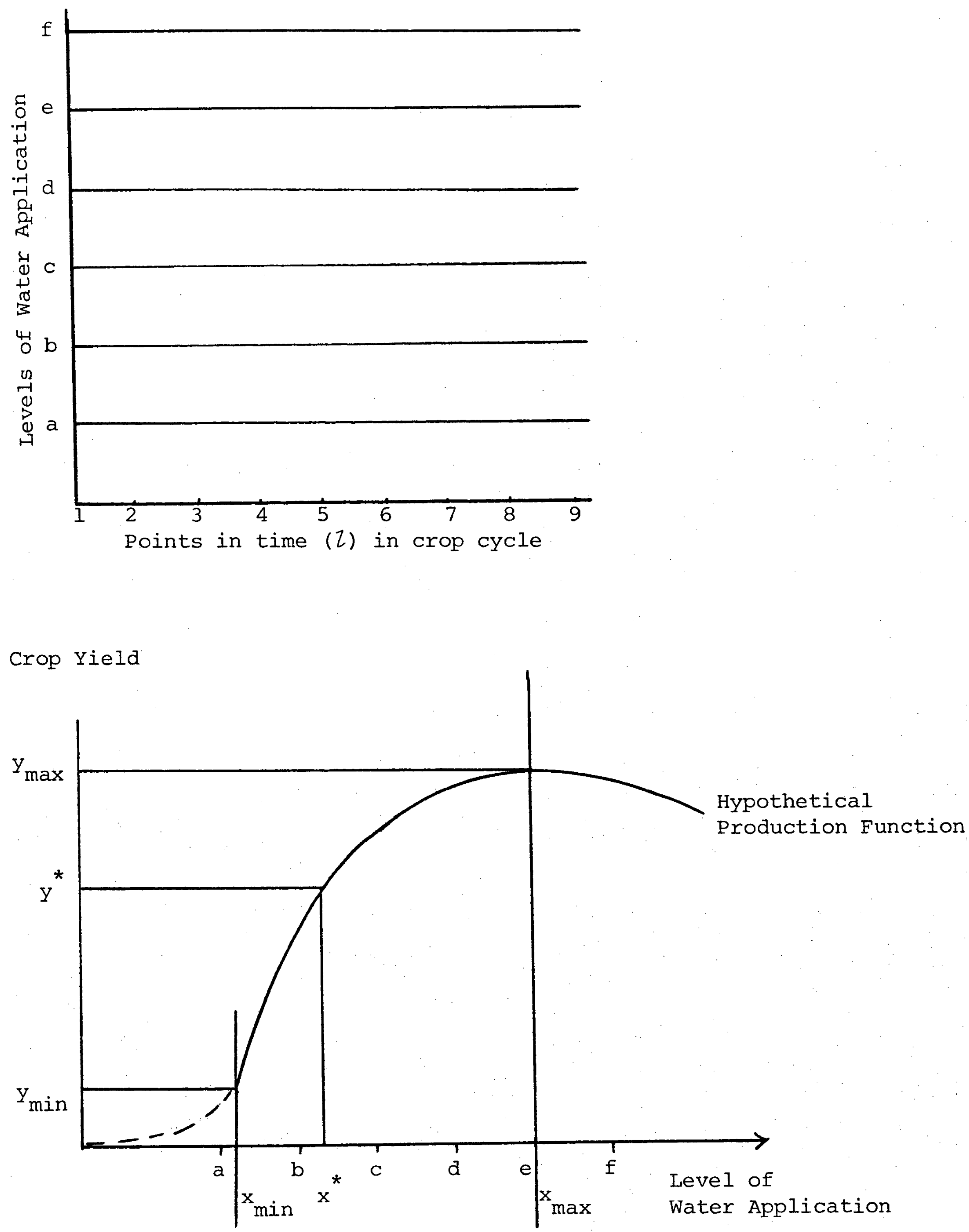
1 It should be clear that it is the *minimum level* of water planned and actually applied to the crop over the season that determines the level of yield. At times, during this period, rainfall may be higher than the *minimum* level, and will help save irrigation even in subsequent stages until the 'soil moisture' draws down to this level.

FIGURE 2.1

A SCHEMATIC REPRESENTATION OF LEVELS OF WATER APPLICATION AND A CONCEPTUAL 'WATER APPLICATION LEVEL - YIELD RESPONSE' FUNCTION FOR EARLY MATURING RICE



x^* - Level of application under our minimal supplementary irrigation policy

x_{max} - Water duty for normal growth

Note also that a minimal supplementation policy is also consistent with maximum conservation of water in the dam.

2.5 Water Resource Optimisation Model for Small Dams

The allocation issue of water resources is complicated because of the existence of another goal. Indulgence in a production goal implies a rapid depletion of storage. It is in conflict with the conservation goal. The choice of the optimum levels of these two conflicting goals for maximum community welfare could be aided by multiple-objective planning procedures. In multiple objective planning, the application of Linear Programming (LP) techniques have been dominant, particularly in relation to water resource problems (Cohon and Marks, 1973; Miller and Byers, 1973).

The optimisation issue in small dams is discussed below within an LP framework.

2.5.1 Hypothetical Trade-off Functions

The problem of optimisation can be formulated in one of the two possible LP formulations, namely, constrained LP and combined LP.

The constrained LP relegates the non-money value objective into the constraint set and only the money value objective is maximised (Cohon and Marks, 1973). In the context of small dams, the problem can be seen as one of maximising the acreage under rice with the preferred conservation level in the constraint set. However, this is not attempted in this dissertation. But, it will be seen that this can be identified as the possible next step to the optimisation problem undertaken here. On the other hand, in the combined LP formulation both the objectives are incorporated into the objective function by a weighting procedure. The first step required in combined LP is the valuation of the non-commensurable objective in money terms. By changing the weights attached to the two objectives, a trade-off function is generated as, for example, demonstrated by Thampapillai and Sinden (1979).

The trade-off function, if generated for the objectives of the small dam problem, would yield a benefit transformation curve of the form shown in Figure 2.2. It should be noted that it is similar to a production possibility curve. Each point on the curve denotes a specific combination of the two goals. The point on the curve at which the benefit transformation curve is tangent to the highest *social indifference curve*[1] denotes the optimal combination at which the community welfare is maximised.

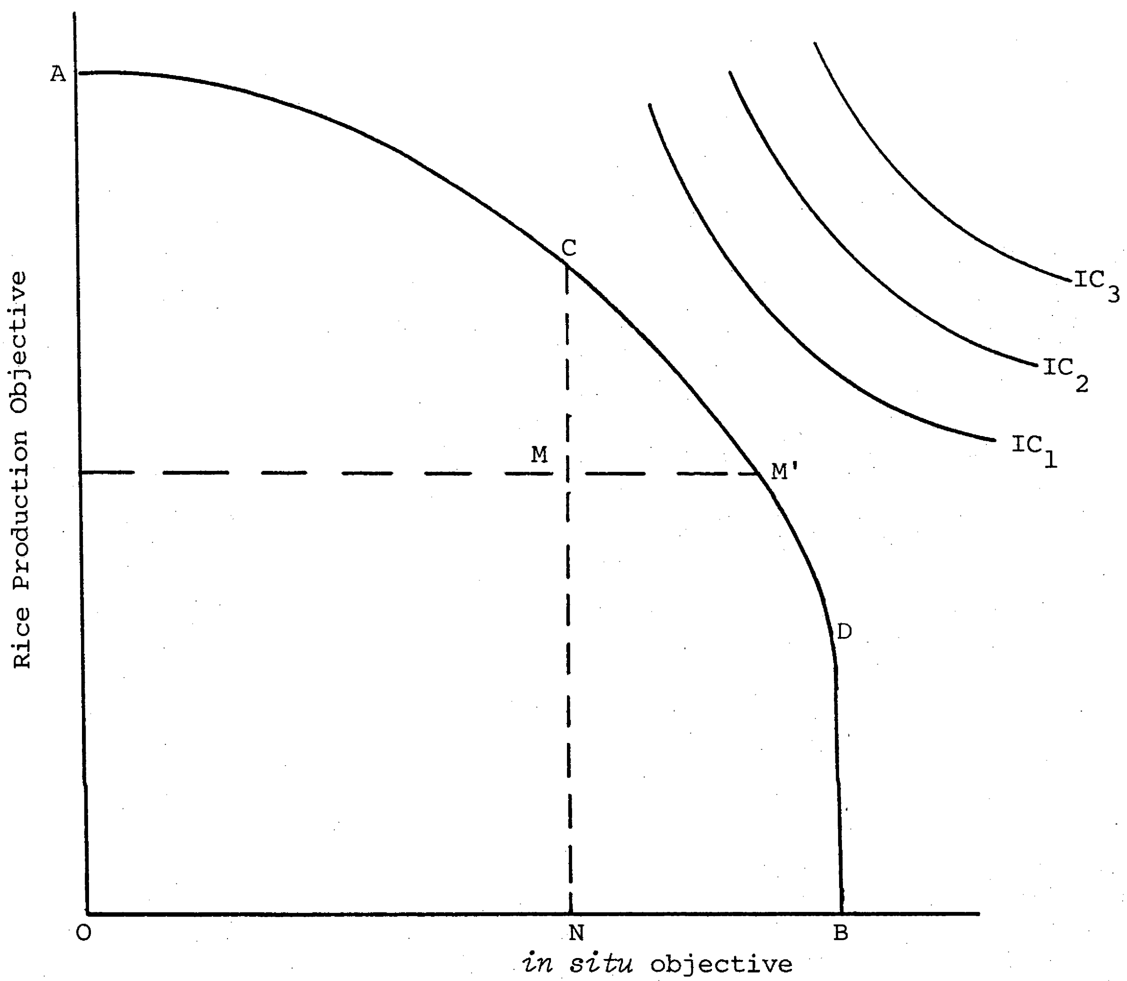
However, the development of a combined LP trade-off function is not our concern. It is also not possible without the levels of *in situ* benefits associated with different levels of dam water storage being valued in money-terms. The relevant difficulties have been mentioned earlier. Nevertheless, in hypothetical form, such a trade-off function offers a useful framework to highlight the nature

1 The set of (convex) indifference curves in Figure 2.2 offer an accepted means of representing the relative ordering of community welfare (or satisfaction), introduced in Sub-Section 2.2.1, in relation to two goals or objectives. Every point on a given curve represents a different combination of the two goals *but* the community is indifferent to the choice of any, since they all provide the *same* level of satisfaction. The further the curve from the origin the higher the level of satisfaction. These are referred to as social indifference curves. A seminal paper is that of Samuelson, P.A. (1956): "Social Indifference Curves", *Quart. Jour. Econ.*, Vol.70, pp.1-22.

FIGURE 2.2

A HYPOTHETICAL BENEFIT TRANSFORMATION FUNCTION

FOR RICE PRODUCTION AND *IN SITU* OBJECTIVES



OB roughly corresponds to amount of *in situ* benefits accruing to the *minimum* preferred level of water storage in the dam, above which no significant additional benefits are derived.

IC_1, IC_2 ... are the social indifference curves.

of the problem undertaken in this research. Looking at Figure 2.2 again, ACM'DB represents the benefit transformation curve. The combination of crop production benefits and *in situ* benefits in the range represented by segment ACM'D are *competetive* with each other. This is because an increase in the crop production will affect the *in situ* benefits and vice versa. Alternatively, segment DB represents a *supplementary* range where increases in water use in crop production will not affect *in situ* benefits, which requires a minimum level of storage in the dam. In fact, OB can be regarded as the near maximum *in situ* benefits associated with the preferred level of water storage. With these basic features identified, the benefit transformation curve can be used to illustrate our problem.

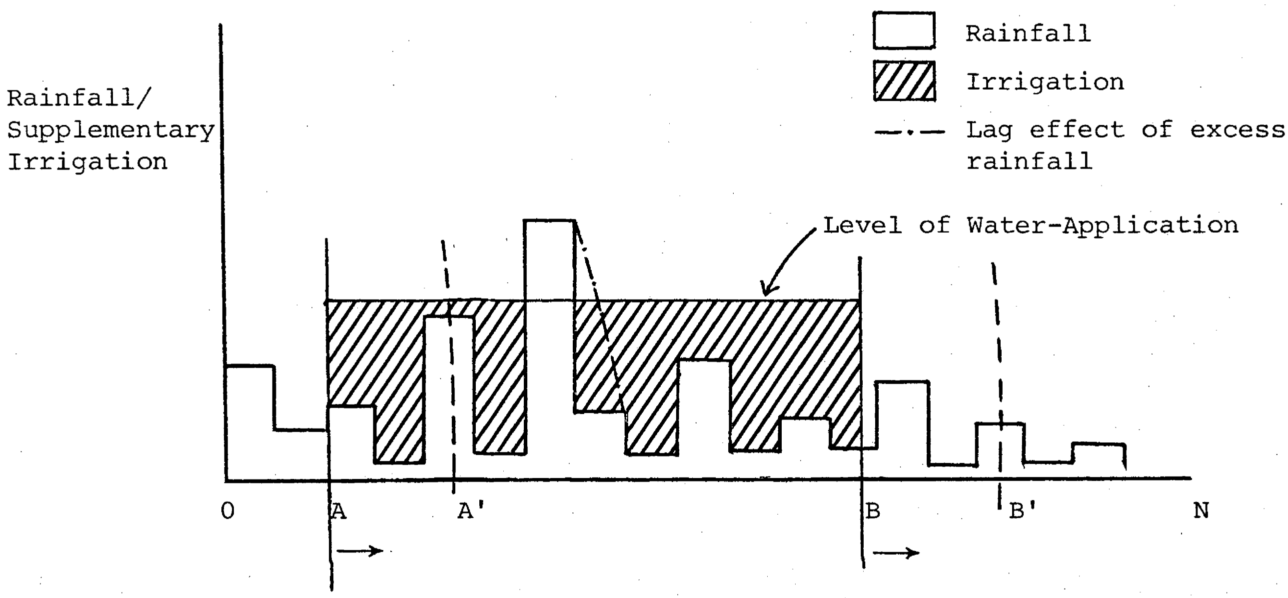
2.5.2 The Specific Optimisation Problem

Given a yield level and associated water duty as, for example, y^* and x^* in Figure 2.1, an obvious procedure for optimising use of the water resource of a dam involves maximum use of the rainfall. This is possible because the rainfall, though stochastic, still has an underlying temporal distribution pattern throughout each year. By adjusting the cropping calendar, therefore, it is possible to utilize the rainfall to maximum advantage and hence to minimise the 'supplementation requirement' via irrigation. Figure 2.3 provides a schematic illustration of adjusting the cropping calendar to match the rainfall distribution.

The possibility of optimisation of water-use can also be illustrated with Figures 2.1 and 2.2. The yield y^* of Figure 2.1 corresponds to the community income NM of Figure 2.2. This income

FIGURE 2.3

AN ILLUSTRATION OF LEVEL OF WATER APPLICATION, RAINFALL DISTRIBUTION AND CROPPING CALENDAR



ON - Complete Dry Season

AB or A'B' - Crop cycle (duration)

level is also associated with a conservation benefit ON. Clearly, point M is inefficient and the movement of M towards M' which is on the frontier will result in an increase in total welfare through additional *in situ* benefits. Theoretically, it is possible to trace all the strategies falling between M and M'.

Alternatively, the different calendar croppings could be treated as alternate processes to achieve the same rice yield (or income). This constitutes the basic feature of a genuine optimisation problem as Koopmans (1977) succinctly stated,

> 'One of the principal elements in the concept of optimality is that the output of one and the same commodity can in general be achieved by more than one process. It is due to this element of choice between alternative ways of achieving the same end result that a genuine optimisation problem arises'.

The different calendar croppings are different processes in that they utilize different combinations of rainfall and irrigation water.

Formally, the optimisation problem can be expressed as a constrained LP problem, but in a slightly different form to the one discussed in Section 2.5.1. The difference arises because of the swapping of places of the two objectives. In the present formulation, the money value objective is relegated to the constraint set. The non-money value benefit, expressed by a proxy, i.e. the water storage at the end of the year, enters the objective function.

Noting again the fact that the maximisation of storage at the end of the year is equivalent to the minimization, Q say, of the total supplementary irrigation, the problem for a given income or production (yield) level, might best be understood in the terminology of Linear Programming, thus:

$$\text{Minimize } Q$$

$$\left.\begin{aligned} &\text{subject to} \quad Q - \sum_{k=1}^{\ell} q_k = 0 \\ &\qquad q_k + u_k \geq b, \; k=1,\ldots,\ell \\ &\qquad q_k, \; u_k \geq 0 \end{aligned}\right\} \tag{2.1}$$

where, q_k and u_k are the irrigation and rainfall at the application time k, and b denotes the fixed intraseasonal water application, required ℓ times during the crop cycle.

The solution procedure of the above problem is not trivial. It requires an iterative approach to arrive at the minimum of Q. The u_k $(k=1,\ldots,\ell)$ sequence have to be picked up from the rainfall calendar each time the Q is computed. The various Q values can then be arranged in an ascending or a descending order. The calendar associated with the minimum Q provides the optimal cropping calendar or period.

The associated water management or withdrawal strategy is efficient, and will be represented by a point, say M', in the Benefit Transformation Curve shown in Figure 2.2. If, at this point, the curve is also tangent to the highest social indifference curve, then the water resource management is optimal. Such an inference of the *global optimum* is not possible without the knowledge of social indifference curves. Our solution is, therefore, at least a *local optimum* or 'efficient allocation point' and can thus narrow the search for the *global optimum*.

2.6 Specific Objectives of the Study

The purpose of the rest of the dissertation is to empirically demonstrate the application of the optimisation model developed in this chapter. The empirical work has two distinct parts. The first part is associated with the development of a formal stochastic water supply model for small dams and the second is concerned with the identification of the cropping calendar associated with a 'local social optimum'. The empirical application utilizes the data from a specific small dam.

More specifically, the objectives are:

(i) to parameterise the dam water storage dynamics primarily in relation to rainfall;

(ii) to elicit and incorporate the stochastic properties and obtain a stochastic simulation model for water storage;

(iii) to develop a 'minimal supplementary irrigation policy' for an early maturing rice crop in small dams;

(iv) to evaluate within the framework provided by (ii), cropping calendars for the irrigation policy defined by (iii) to determine the optimal one; and

(v) to examine the validity of the results of (iv) in the variable rainfall environment making use of many years' rainfall data.

The analysis with respect to the last three objectives rests upon the development of a stochastic dynamic water supply model as specified in (i) and (ii). In the following chapter, the methodology for parameterising the water storage dynamics and developing a stochastic simulation model for water supply is presented.

CHAPTER 3

METHODOLOGY: TIME SERIES ANALYSIS AND STOCHASTIC SIMULATION

Chapter 2 concluded by pointing out the need for a stochastic dynamic model of water supply for the solution of the optimisation problem (2.1). This chapter is devoted to invoking a systems methodology for parameterising the dynamic water storage behaviour and developing a stochastic simulation model. Section 3.1 defines system concepts and deals with the specification of a Transfer Function (TF) model for rainfall-water storage. A cursory note on the conventional approach to parameterising water storage is also included. For identification and estimation of the TF model, the basis of a recursive time series analysis approach is presented in Section 3.2. This provides a detailed account of the Instrumental Variable - Approximate Maximum Likelihood (IV-AML) technique, adopted in this research, the underlying assumptions made and the computer package 'CAPTAIN' available for its implementation. Section 3.3 deals with the development, based on the IV-AML estimates, of a stochastic - Monte Carlo simulation model for water storage, and outlines the approach to the analysis of the withdrawal strategies for optimisation of the water resource.

3.1 Parameterisation: Transfer Function Model

Natural water storage in small dams is a changing or dynamic process with rainfall, *especially* its intensity and duration throughout the year, being the obvious major causal factor. To date, however, no concerted attempt has been made to establish causality or to parameterize the storage behaviour of small dams in Sri Lanka. In fact, the associated difficulties have relegated the water resource problem, in spite of its central importance, to a subsidiary role in discussions of physical resources. However, it must be conceded that the need for forecasting or prediction of storage is now gaining acceptance. For example, in a paper primarily addressed to the small dam, Somasiri (1976, p.87) states:

> '... a good understanding of the water resources, ability to forecast the tank storage by estimating all the gains and losses are essential for the preparation of cropping programmes.'

Subsequently, there have been piecemeal attempts to gain this understanding: for example, instrumentation to measure certain hydrological aspects such as evaporation and outflow through the sluice. Evidently, the conceptual basis of such attempts[1] has evolved from hydrology where a 'water-balance' concept has predominated as evinced by Somasiri (1976). This concept embodies the actual measurements of all forms of gains to and losses from the dam storage so as to infer the net or 'balance' availability.

A water-balance approach is deterministic and its forecasting ability is limited, especially in a variable rainfall environment. Year to year variability in rainfall distribution is likely to result in variable gains to storage at any given time of the year. This type of deterministic forecasting can, therefore, be not only misleading but also disastrous when it is used for

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1 Department of Agricultural Engineering, University of Peradeniya collaborated with the Government Department of Agriculture in 1978 in monitoring a few aspects in the same dam with which this dissertation is specifically concerned.

crop planning for the dry season. To be useful, such forecasts need to consider many years of observations so that probable ranges of values of the various components can be understood. This will involve extensive instrumentation to collect accurate data on a continuous basis for a reasonably long period of time. The need for extensive data for many elements of the hydrology exacerbates the problem.

As we shall see, an effective yet simple approach is provided by the application of simple system concepts to describe the dominant characteristics of our dynamic problem and by the addition of a stochastic (or probabilistic uncertainty) component to complement and quantify the limitations of this simplicity.

3.1.1 System Concepts

Fundamentally, the 'formal' system philosophy maintains that any portion of the real world such as, for example, the catchment - water storage phenomenon, can be viewed as a *system* (Bennet and Chorley, 1978). In general an environmental system, such as the one under consideration, manifests the inter-relationships of three elements, namely, input (or causal variable), output (or response variable) and throughput (or the transfer characteristics). In accordance with Bennet and Chorley (1978) a 'formal' system can be defined as:

> a set of logical operations acting upon, and acted upon by, one or more inputs. These inputs lead to the production of outputs from the system and the process of throughput is capable of sustaining the operational structure and identity of the system.

In effect, the throughput is the operator that links the input with the output. Such a system is capable of specification, analysis and control in a more or less rigorous manner. Partially for this reason, such a system is referred to as a *hard* system.

Formally, the operation of the system with the input, u_k and output, x_k at time step k can be represented by the transformation equation:

$$x_k = S\, u_k$$

where the operator 'S' is referred to as the *system transfer function* (TF). It is this element that determines the way in which the input, u_k is translated to become the output, x_k at a given time k. Hence, the TF is unique to a system and therefore, its characterization also helps to define the system itself. Usually, the TF has a specific structure and is made up of a number of parameters which determine the magnitude and form of modulation induced on the input by the system at the output.

Inputs in systems are usually categorized into many groups: for example, transient impulse, unit step, sinusoidal, ramp and stochastic (Bennet and Chorley, 1978). In many realistic systems, the input can be considered as a combination of these forms. Of these, the transient impulse input is of most interest in the water-storage system. A transient impulse input represents a point stimulus into the system and is only momentary. In continuous systems, impulsive inputs can be represented by the delta function $\delta(k)$, which is defined as:

$$\delta(k) = \begin{cases} 1 & k = \varepsilon \\ 0 & -\infty < k < \infty,\ k \neq \varepsilon \end{cases}$$

A rainstorm of very short duration in a catchment can be considered as an impulse input to a hydrologic system.

3.1.2 The Impulse Response Function (IRF) of Hydrologic Systems

The output that a system yields in response to a sequence of inputs, say impulses, comprises an aggregate picture. Distinguishing the effects of any single impulse input on the overall output, however, is not straightforward. This is mainly because the responses to inputs at one time have not decayed away completely before the impulse at the next time begins to produce an effect. This is particularly true of systems with frequently recurrent impulse inputs. In contrast, the overall output response for a sequence of impulses can be specified when the *impulse response function* (IRF) is known.

The output or response of a system to a single unit impulse input of unit duration[1] is referred to as the impulse response function (IRF). Thus, to allude to our previous example, the changing pattern of runoff levels with time in a catchment following a rainstorm of short duration represents the IRF of one hydrologic system and it is known as the 'unit hydrograph'. Such a pattern in this case would involve a rise followed by an exponential type decay. The IRF can vary depending upon the magnitude of inputs in non-linear systems such as, for example, the watershed runoff response to rainfall input. Perhaps this is so for many hydrologic systems. Whereas in a linear system, the IRF remains the same irrespective of the level of input. This invariance property of IRF's in a linear system is also known as *additivity* or the *law of superposition.* This is because the response of a sum of impulse inputs is the same as the sum of the impulse response functions for each of the individual inputs. Often non-linear systems, including

1 We are now considering discrete-time systems where an impulse is active over the entire sampling interval which is the observation period divided by N - 1, N being the number of samples taken.

our water storage system, can be linearised. The probable hydrologic causes of non-linearity in our system and an appropriate procedure of 'linearisation' are detailed in Chapter 4. For the moment, it will suffice to say that a linear approximation involving the characterisation of an 'effective' rainfall measure is possible for this problem and, perhaps, for many hydrologic systems. Henceforth discussions in this chapter conveniently assume a linear or linearised hydrologic system.

The hydrologic system is also dynamic since it receives quantitative inputs, say rainstorms, which vary in time and act consistently under given constraints to yield quantitative outputs. These also vary in time in a dynamic manner governed by the transfer characteristics. An adequate approximation of the behaviour of such a dynamic linear system can be represented by the linear filter of the form:

$$x_k = g_o u_k + g_1 u_{k-1} + \dots + g_\infty u_{k-\infty}$$

Clearly, this is an infinite dimensional discrete time representation. More importantly, it should be noted that the weights: $g_o, g_1, \dots, g_\infty$ constitute the IRF. Its mathematical basis is shown elsewhere. The 'formal' IRF provides a useful concept for understanding the behaviour of the overall output response.

As mentioned earlier, for a given sequence of input signals, the overall output response can be visualized as the aggregate manifestation of a series of superimposed IRFs. Of course, each IRF needs an arithmetic scaling up or down depending upon the magnitude of the input. Such a behaviour of overall output is referred to as *convolution* (Bennet and Chorley, 1978). It is, in fact, the property of convolution of linear dynamic systems that provides the basis for the formalisation of the linear systems approach to our water storage problem. The conceptual convolution property is illustrated in Figure 3.1(a) to (d). This shows the sequence moving from an impulse in (a) to its output response in (b) to the superimposed output responses of a series of impulses (c) and, finally, to the overall output of the system (d).

3.1.3 Impulse Response Function (IRF) and Transfer Function (TF): the Mathematical Basis

Let us consider a convolution integral equation of the form:[1]

$$x(t) = \int_{o}^{t} r(t-w)\ u(w)\ dw \tag{3.1}$$

This represents a noise-free linear dynamic system with continuous input, u(t) and output, x(t) at time t. In other words, it describes a system, where an input, u(t) is convoluted with the IRF, r(t) to yield an output x(t). In our water storage problem, the kernel r(t) is the 'storage impulse response function', which we have to determine from data on the system.

A numerical procedure for solving the integral equation (3.1) is based on mathematical transformation and approximation.

---

1 The general representation of a linear integral equation of the *first kind* is of the form:

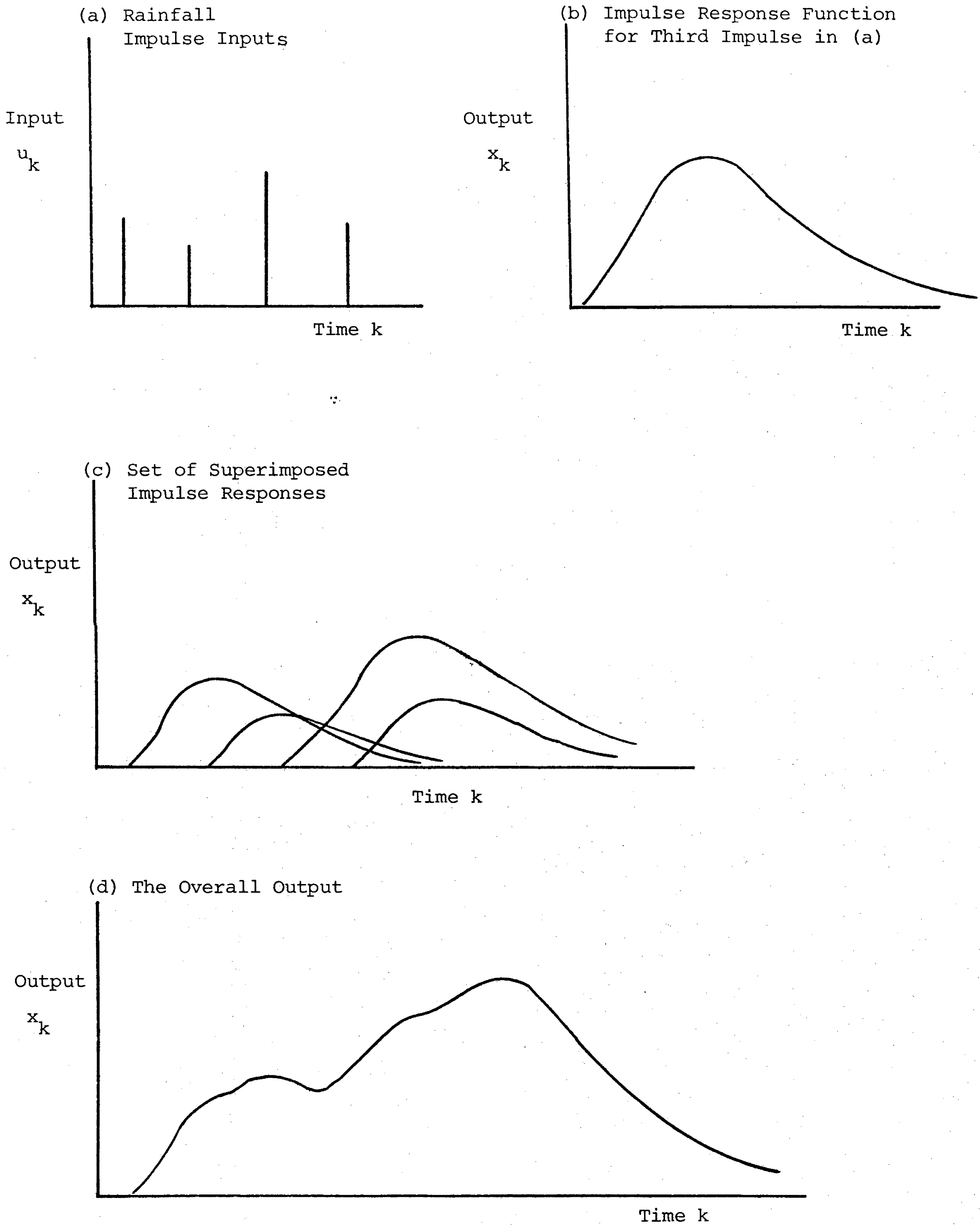
$$x(t) = \int r(t,w)\ u(w)\ dw$$

It is said to be *convolution* when the kernel has the property: r(t,w) = r(t-w).
There are many practical examples of systems which exhibit this property (Jakeman and Young, 1980).

FIGURE 3.1

SCHEMATIC REPRESENTATION OF CONVOLUTION

OF EFFECTIVE RAINFALL



Source: Adapted from Bennet and Chorley (1978), p.37.

The Laplace transform[1] $L\{x(t)\}$ of $x(t)$ of the convolution integral equation (3.1) is given as:

$$X(s) = R(s)\ U(s)$$

In practice, $R(s)$ can be well approximated (see Takahashi et al, 1972) by a ratio of polynomials[2] $\frac{B(s)}{A(s)}$ in the Laplace operator s where[3]

$$B(s) \triangleq b_o + b_1 s + b_2 s^2 + \ldots + b_m s^m$$

and

$$A(s) \triangleq 1 + a_1 s + a_2 s^2 + \ldots + a_n s^n$$

In this way, our transformed equation can be rewritten as:

$$A(s)\ X(s) = B(s)\ U(s) \tag{3.2}$$

Equation (3.2) can be inverted back from the Laplace domain to the original time domain by taking inverse Laplace transforms. Then we obtain the continuous time ordinary differential equation of the form:

$$A(D)\ x(t) = B(D)\ u(t) \tag{3.3}$$

where D is the differential operator. On expansion equation (3.3) becomes:

$$\begin{aligned} 1 + a_1 \frac{dx(t)}{dt} + \ldots + a_n \frac{d^n x(t)}{dt^n} &= b_o u(t) \\ &+ b_1 \frac{du(t)}{dt} + \ldots + b_m \frac{d^m u(t)}{dt^m} \end{aligned} \tag{3.4}$$

A discrete-time version of equation (3.4) is a more relevant one in the present study, since we are dealing with discrete sampled data.

1 The Laplace transform $F(s)$ of a continuous function $f(t)$ is given by:

$$F(s) = \int_0^\infty e^{-st} f(t)\ dt$$

2 Of course, they can be exactly equal.

3 The symbol $\triangleq$ means 'is defined as'.

The equivalent difference equation representation for N input-output samples is as follows:

$$x_k + a_1 x_{k-1} + \ldots + a_n x_{k-n} = b_o u_k + b_1 u_{k-1} + \ldots + b_m u_{k-m}$$

$$(k=1,2,\ldots,N) \tag{3.5}$$

where the values of m and n may be different to those of equation (3.4) and the values of a_i and b_i are almost certainly different.

Making use of the backward shift operator[1] (z^{-1}) this equation can be written as:

$$x_k = \frac{B(z^{-1})}{A(z^{-1})} u_k \tag{3.6}$$

where

$$A(z^{-1}) \triangleq 1 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_n z^{-n}$$

and

$$B(z^{-1}) \triangleq b_o + b_1 z^{-1} + b_2 z^{-2} + \ldots + b_m z^{-m}$$

In practice, the polynomials $A(z^{-1})$ and $B(z^{-1})$ are small, for example, of the order of one to five parameters. Thus the transfer function 'S' introduced earlier is now approximated by two low dimensional polynomials. For this reason, such a representation is considered *parsimonious* or parametrically efficient. This is discussed in detail by Box and Jenkins (1970).

Further, by dividing $A(z^{-1})$ into $B(z^{-1})$, it is possible to obtain the infinite dimensional representation of the original convolution integral such that:

$$\frac{B(z^{-1})}{A(z^{-1})} = g_o + g_1 z^{-1} + \ldots + g_\infty z^{-\infty}$$

$$\triangleq G\ (z^{-1}) \tag{3.7}$$

This is in fact the IRF.

---

1 In backward shift operator notation

$$x_{k-\ell} = (z^{-\ell})\ x_k \quad , \ \ell = o,1,\ldots$$

Equations (3.6) and (3.7) provide two alternative ways of representing linear systems. Models based on the infinite dimensional representation are known as *weighting sequence* models, whilst those of the form of equation (3.6) are popularly known as *Transfer Function*[1] (TF) models. Studies in hydrology based on the former are relatively few; for example, see Natale et al (1974) and Szollosi-Nagy (1976).

In the present study, to model water storage a TF model is favoured, since it is as general a representation as the other but it also has the important advantage of parametric efficiency. Such a property is desirable because it reduces parameter uncertainty or variance when estimation of the model parameters is invoked.

3.1.4 Specification of Transfer Function (TF) Model for Water Storage

To this point, attention has been concentrated on linear systems whose inputs and outputs are exactly measured and to which there are no outside disturbances. In order to account for 'errors in variables'[2] and other disturbances in TF models, a lumped noise disturbance, ξ_k can be added to the hypothetical noise-free output x_k to yield the observed output, y_k. The complete model and its components are schematically shown in Figure 3.2. This is the same as that used by Box and Jenkins (1970) and Young (1971,1974). It must be noted that the noise term, ξ_k is assumed to have a rational spectral density. In other words, it is considered to be the output

---

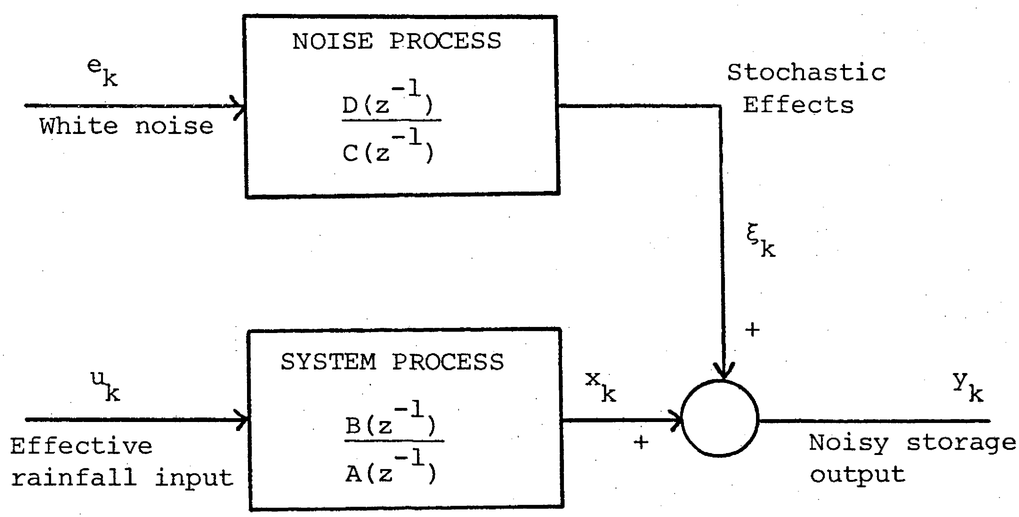
1 Transfer Function is also called a Rational Distributed Lag Function (Jorgenson, 1966).

2 See page 63 and the reference to Kendall and Stuart (1961).

FIGURE 3.2

EFFECTIVE RAINFALL - WATER STORAGE

TIME-SERIES MODEL



Source: Adapted from Young (1976), p.596.

of a TF whose input, e_k is a zero mean, serially uncorrelated sequence of random variables with variance σ^2; i.e.,

$$E\{e_k\} = 0, E\{e_j e_k\} = \sigma^2 \delta_{jk} \tag{3.8}$$

However, this structural assumption on ξ_k is not necessary for adequate estimation of the system TF model parameters in (3.5) but, as we shall see, proves useful for our later simulation purposes.

With this assumption, the conventional statistical terminology is as follows: ξ_k can be regarded as generated by

Autoregressive Moving Average (ARMA) process acting on the white noise input e_k, (see, for example, Box and Jenkins, (1970).[1]

After having incorporated the noise term, the various relationships between the variables u_k, x_k, ξ_k and y_k can again be rewritten as follows:

$$A(z^{-1})\ x_k = B(z^{-1})\ u_k$$

$$C(z^{-1})\ \xi_k = D(z^{-1})\ e_k$$

$$y_k = x_k + \xi_k$$

or equivalently,

$$y_k = \frac{B(z^{-1})}{A(z^{-1})}\ u_k + \frac{D(z^{-1})}{C(z^{-1})}\ e_k \tag{3.9)a}$$

where $C(z^{-1})$ and $D(z^{-1})$ are also polynomials of the form:

$$C(z^{-1}) \triangleq 1 + c_1 z^{-1} + \ldots + c_p z^{-p}$$

$$D(z^{-1}) \triangleq 1 + d_1 z^{-1} + \ldots + d_q z^{-q}$$

The above TF model can also be specified in an alternative vector-matrix form which is more useful for deriving algorithms for the estimation of the unknown parameters:

$$\begin{aligned} y_k &= \underset{\sim}{z}_k^T\ \underset{\sim}{a} + \eta_k \\ \xi_k &= \underset{\sim}{v}_k^T\ \underset{\sim}{c} + e_k \end{aligned} \tag{3.9)b}$$

---

1 In $C(z^{-1})\ \xi_k = D(z^{-1})\ e_k$, for instance: the polynomials $C(z^{-1})$ and $D(z^{-1})$ represent the AR and MA components of order, say, p and q respectively. The model is usually represented as ARMA (p,q).

where

$$\eta_k \triangleq a_1\xi_{k-1} + \ldots + a_n\xi_{k-n} + \xi_k$$

$$\underset{\sim}{z}_k^T \triangleq [-y_{k-1}, \ldots, -y_{k-n}, u_k, \ldots, u_{k-m}]$$

$$\underset{\sim}{a} \triangleq [a_1, \ldots, a_n, b_o, \ldots, b_m]^T$$

$$\underset{\sim}{v}_k^T \triangleq [-\xi_{k-1}, \ldots, -\xi_{k-p}, e_{k-1}, \ldots, e_{k-q}]$$

$$\underset{\sim}{c} \triangleq [c_1, \ldots, c_p, d_1, \ldots, d_q]^T$$

The model specified by the equation (3.9) contains parameters in the $\underset{\sim}{a}$ and the $\underset{\sim}{c}$ vectors that characterize the system model and the noise model, respectively, and are not exactly known beforehand. The estimation problem is then to use the sampled effective rainfall input data, u_k and storage output data y_k: first, to *identify* the number of parameters in $\underset{\sim}{a}$ (n and m) and $\underset{\sim}{c}$ (p and q) that characterize the model; next, to obtain consistent[1] estimates of the parameters.

3.2 Recursive Time Series Analysis

The estimation of TF models falls within the gamut of time series analysis. Many techniques are now available for use and the interested reader is referred to Astrom and Eykhoff (1971) for a survey. Perhaps the approach propounded and used by Box and Jenkins (1970) is best known. Another is a recursive instrumental variable (IV) approach, for which Young (1971; 1972) is possibly the foremost

---

1 Consistency is an asymptotic (or large sample) property of the estimator. For example, $\hat{a}_k$ is a consistent estimator of an element 'a' in the parameter vector if the probability limit of $\hat{a}_k$ is a. Strictly speaking, $\hat{a}_k$ converges to a in the probability limit, if for any $\delta > 0$,

$$\lim_{k \to \infty} \text{Prob } (|a-\hat{a}_k| < \delta) = 1$$

See Pindyck and Rubinfeld (1976), p.23.

proponent; broadly on the grounds that it is simple, effective and extremely robust in application to many forms of noise on the system (Young and Jakeman, 1979a). It does not require the rational spectral density assumption (that is, ARMA) on the noise to obtain consistent estimates.[1] The recursive procedure for estimating the noise model when invoked is known as Approximate Maximum Likelihood (AML). Like the IV procedure, it has an inherent potential for updating parameter estimates while passing through the data serially.

For the estimation of the storage TF model, the present study employs the recursive IV-AML technique. Many of the advantages that led to this choice stem from its recursive nature. The basis of this technique can be derived from an analogy with a vector-matrix formulation of the Ordinary Least Squares (OLS) analysis. This is well expounded, for example, by Young (1972; 1974). However, in general, applied economists seem to be unacquainted with this technique and, therefore, it is felt that a brief outline is essential to highlight the procedure and its value.

3.2.1 Recursive Least Squares (RLS)

The OLS solution for linear regression problems can easily be formulated in recursive terms. Let us consider the multiple linear regression model:

$$y_i = \underset{\sim}{x}_i^T \underset{\sim}{a} + \varepsilon_{yi} \qquad (i=1,\ldots, k) \tag{3.10}$$

where

$$\underset{\sim}{x}_i^T = [x_{1i}, x_{2i}, \ldots, x_{ni}],$$

the x_{li}'s being n exactly known linearly independent variables at the i-th time-step (sample);

$$\underset{\sim}{a} = [a_1, a_2, \ldots, a_n]^T$$

1 However, it can be modified to yield statistically efficient estimates if the noise does possess ARMA characteristics; see Young (1976), Young and Jakeman (1979a), as pointed out later (page 67).

and y_i is the output sequence corrupted by error ε_{yi}.

The least squares criterion for the solution of the parameters in $\underset{\sim}{a}$ is given as:

$$J \triangleq \sum_{i=1}^{k} (\underset{\sim}{x}_i^T \underset{\sim}{a} - y_i)^2 \tag{3.11}$$

Setting all the partial derivatives of J with respect to each of the elements of $\underset{\sim}{a}$ simultaneously to zero, yields a set of normal equations:

$$\frac{\partial J}{\partial \underset{\sim}{a}} = [\sum_i \underset{\sim}{x}_i \underset{\sim}{x}_i^T] \underset{\sim}{a} - \sum_i \underset{\sim}{x}_i y_i = \underset{\sim}{0} \tag{3.12}$$

where $\frac{\partial J}{\partial \underset{\sim}{a}}$ represents the gradient of J with respect to $\underset{\sim}{a}$.

The solution of the set of equations (3.12) is of the form:

$$\hat{\underset{\sim}{a}}_k = P_k \underset{\sim}{b}_k \tag{3.13}$$

where $P_k \triangleq [\sum_{i=1}^{k} \underset{\sim}{x}_i \underset{\sim}{x}_i^T]^{-1}$ and $\underset{\sim}{b}_k \triangleq \sum_{i=1}^{k} \underset{\sim}{x}_i y_i$

Here $\hat{\underset{\sim}{a}}_k$ represents the estimate of parameters after k samples.

In a recursive formulation, $\hat{\underset{\sim}{a}}_k$ could be represented as a linear sum of the estimate obtained after (k-1) samples, $\hat{\underset{\sim}{a}}_{k-1}$, plus a corrective term. The corrective term is based on the information y_k and $\underset{\sim}{x}_k$ received at the k-th sampling instant (or k-th observations). It can be noted from equation (3.13) that:

$$P_k^{-1} = P_{k-1}^{-1} + \underset{\sim}{x}_k \underset{\sim}{x}_k^T \tag{3.14}$$

and that

$$\underset{\sim}{b}_k = \underset{\sim}{b}_{k-1} + \underset{\sim}{x}_k y_k \tag{3.15}$$

By straightforward matrix manipulations, as shown by Young (1972), equation (3.14) can be easily transformed into the following recursive expression for P_k:

$$P_k = P_{k-1} - K_k \underset{\sim}{x}_k^T P_{k-1} \tag{3.16}$$

where K_k is a gain vector and

$$K_k = P_{k-1} \underset{\sim}{x}_k [1 + \underset{\sim}{x}_k^T P_{k-1} \underset{\sim}{x}_k]^{-1}$$

The recursive equation for $\hat{\underset{\sim}{a}}_k$ in terms of $\hat{\underset{\sim}{a}}_{k-1}$ can then be obtained by substituting from equations (3.15) and (3.16) into equation (3.13) as

$$\hat{\underset{\sim}{a}}_k = \hat{\underset{\sim}{a}}_{k-1} - K_k \{\underset{\sim}{x}_k^T \hat{\underset{\sim}{a}}_{k-1} - y_k\} \tag{3.17}$$

Or equivalently,

$$\hat{\underset{\sim}{a}}_k = \hat{\underset{\sim}{a}}_{k-1} - P_k \{\underset{\sim}{x}_k \underset{\sim}{x}_k^T \hat{\underset{\sim}{a}}_{k-1} - \underset{\sim}{x}_k y_k\} \tag{3.18}$$

Notably, this derivation is completely deterministic having not taken into account the nature of the error ε_{yi}. However, by making assumptions as in equation (3.8) about the errors, it can be shown that the parameter estimates are consistent[1] and that the estimation error vector, $\tilde{\underset{\sim}{a}}_k = \hat{\underset{\sim}{a}} - \underset{\sim}{a}$ exhibits the following asymptotic statistical properties:

(a) zero mean value; and

(b) the relationship of the variance - covariance matrix, P^* to the P matrix (P_N after N sample) is of the form:

$$P^* = \sigma^2 P \tag{3.19}$$

where

$$P^* \triangleq E\{\tilde{\underset{\sim}{a}}_k \tilde{\underset{\sim}{a}}_k^T\}$$

1 See the footnote on p.58.

Ultimately, the true recursive least squares (RLS) estimates and their variance-covariance matrix obtained by substitution are as follows:

$$\hat{\underset{\sim}{a}}_k = \hat{\underset{\sim}{a}}_{k-1} - \frac{P_k^*}{\sigma^2} \{\underset{\sim}{x}_k \underset{\sim}{x}_k^T \hat{\underset{\sim}{a}}_{k-1} - \underset{\sim}{x}_k y_k\}$$

and

$$P_k^* = P_{k-1}^* - P_{k-1}^* \underset{\sim}{x}_k \{\sigma^2 + \underset{\sim}{x}_k P_{k-1}^* \underset{\sim}{x}_k^T\}^{-1} \underset{\sim}{x}_k^T P_{k-1}^*$$

The RLS can thus provide an on-line[1] estimate of parameters at each sampling instant along with a P^* covariance matrix. Other important advantages of a recursive approach include:

(i) the provision of information on the convergence of parameters; a lack of convergence may be associated with system non-linearity or non-stationarity;

(ii) it can be very simply modified so that the time variation of the non-converging parameters can be monitored (Young, 1974). In this way, inference of the cause of the non-stationarity or non-linearity, for example, is aided by observation of the pattern of temporal variation (Young, 1978); and

(iii) a matrix inversion is not required at each updating of parameter values, since $\{\sigma^2 + \underset{\sim}{x}_k P_{k-1}^* \underset{\sim}{x}_k^T\}$ is a scalar; this can provide tremendous savings on computer space.

---

1 That is, data can be fed directly into a computer as they become sequentially available and the parameter estimates can be provided in real time.

While the RLS has nice properties, like (a) and (b) mentioned in this section, when applied to regression models of the form of equation (3.10), it has limitations for direct application to so called *structural* models[1] like the TF model. It can be noted that the $\underset{\sim}{z}_k^T$ vector in the equation (3.9)b contains lagged dependent variables in contrast to the corresponding $\underset{\sim}{x}_i^T$ vector in equation (3.10), which only contains independent variables. However, the RLS can be regarded as a useful building block for the more sophisticated yet still simple recursive IV-AML technique.

3.2.2 The Instrumental Variable - Approximate Maximum Likelihood (IV-AML) Technique

The Instrumental Variable - Approximate Maximum Likelihood (IV-AML) algorithms involve only simple modifications to the least squares solution. Instrumental variable techniques are well known in the non-recursive statistical literature dating back to Riersol (1941). Roughly, an instrumental variable is defined as one that is uncorrelated with the residual or noise and highly correlated with the independent variable (Kendall and Stuart, 1961). As we shall see, it is used in the IV-technique, as in conventional statistics, to obviate the problem of noise induced asymptotic bias[2] on structural

1 Both input and output sequences are stochastic in *structural* models, in contrast to *regression* models where only the output is stochastic while the input is deterministic and often predetermined (Kendall and Stuart, 1961, pp.392-93).

2 Asymptotic unbiasedness is defined as follows: If $\hat{a}_k$ is an estimator of 'a' based on a sample size k, $\hat{a}_k$ is called an asymptotically unbiased estimator of 'a' if:

$$\lim_{k \to \infty} E\{\hat{a}_k\} = a.$$

A more rigorous definition to cover even situations where $E\{\hat{a}_k\}$ does not exist, is given by Hood and Koopmans (1953) as follows: $\hat{a}_k$ is an asymptotically unbiased estimator for 'a', if the mean of the limiting distribution of $\sqrt{k}\ (\hat{a}_k - a)$ is zero.

models like the TF model.

The need for IV arises since the noise in the specified model is not independent. Recall that the specified storage model is:

$$y_k = \underset{\sim}{z}_k^T \underset{\sim}{a} + \eta_k$$

where

$$\underset{\sim}{z}_k^T = [-\ y_{k-1}, \ldots, -\ y_{k-n}, u_k, \ldots, u_{k-m}]$$

and

$$\eta_k = a_1 \xi_{k-1} + \ldots + a_n \xi_{k-n} + \xi_k$$

Clearly, in this relationship, the noise, η_k and $\underset{\sim}{z}_k$ are not independent. In practice, consistent estimates can be obtained by the use of *an* IV-vector $\hat{\underset{\sim}{x}}_k$ such as:

$$\hat{\underset{\sim}{x}}_k \triangleq [-\ \hat{x}_{k-1}, \ldots, -\hat{x}_{k-n}, u_k, \ldots, u_{k-m}]$$

and the $\hat{x}_k$'s are the best estimates available of the noise-free portion x_k of the noisy output $y_k = x_k + \xi_k$.

For the modified problem, the non-recursive solution can then be given as follows (see Young, 1974):

$$\hat{\underset{\sim}{a}}_k = \hat{P}_k \hat{\underset{\sim}{b}}_k$$

where $\hat{P}_k = \left[\sum_{i=1}^{k} \hat{\underset{\sim}{x}}_i \underset{\sim}{z}_i^T\right]^{-1}$ and $\hat{\underset{\sim}{b}}_k = \sum_{i=1}^{k} \hat{\underset{\sim}{x}}_i y_i$

This is similar in structure to that presented for the OLS solution (3.1 to the regression problem. Adopting a procedure similar to the one used for RLS, as shown by Young (1972; 1974), the recursive solution can be obtained as

$$\hat{\underset{\sim}{a}}_k = \hat{\underset{\sim}{a}}_{k-1} - \hat{K}_k \{\underset{\sim}{z}_k^T \hat{\underset{\sim}{a}}_{k-1} - y_k\}$$

$$\hat{P}_k = \hat{P}_{k-1} - \hat{K}_k \underset{\sim}{z}_k^T \hat{P}_{k-1} \tag{3.20}$$

$$\hat{K}_k = \hat{P}_{k-1} \hat{\underset{\sim}{x}}_k [1 + \underset{\sim}{z}_k^T \hat{P}_{k-1} \hat{\underset{\sim}{x}}_k]^{-1}$$

It should be noted that the biased RLS solution for our structural model would have had $\underset{\sim}{z}_k$ wherever $\hat{\underset{\sim}{x}}_k$ occurs.

It now remains to discuss an appropriate practical means of generating $\hat{\underset{\sim}{x}}_k$. Updating is accomplished using an *auxiliary model* of the system (see Figure 3.3). Thus, if $\hat{\underset{\sim}{a}}$ is the best current estimate of $\underset{\sim}{a}$ then $\hat{\underset{\sim}{x}}_k$ is obtained from (c.f. (3.5)):

$$\hat{x}_k = - \hat{a}_1 \hat{x}_{k-1} - \dots - \hat{a}_n \hat{x}_{k-n} + \hat{b}_o u_k + \dots + \hat{b}_m u_{k-m} \tag{3.21}$$

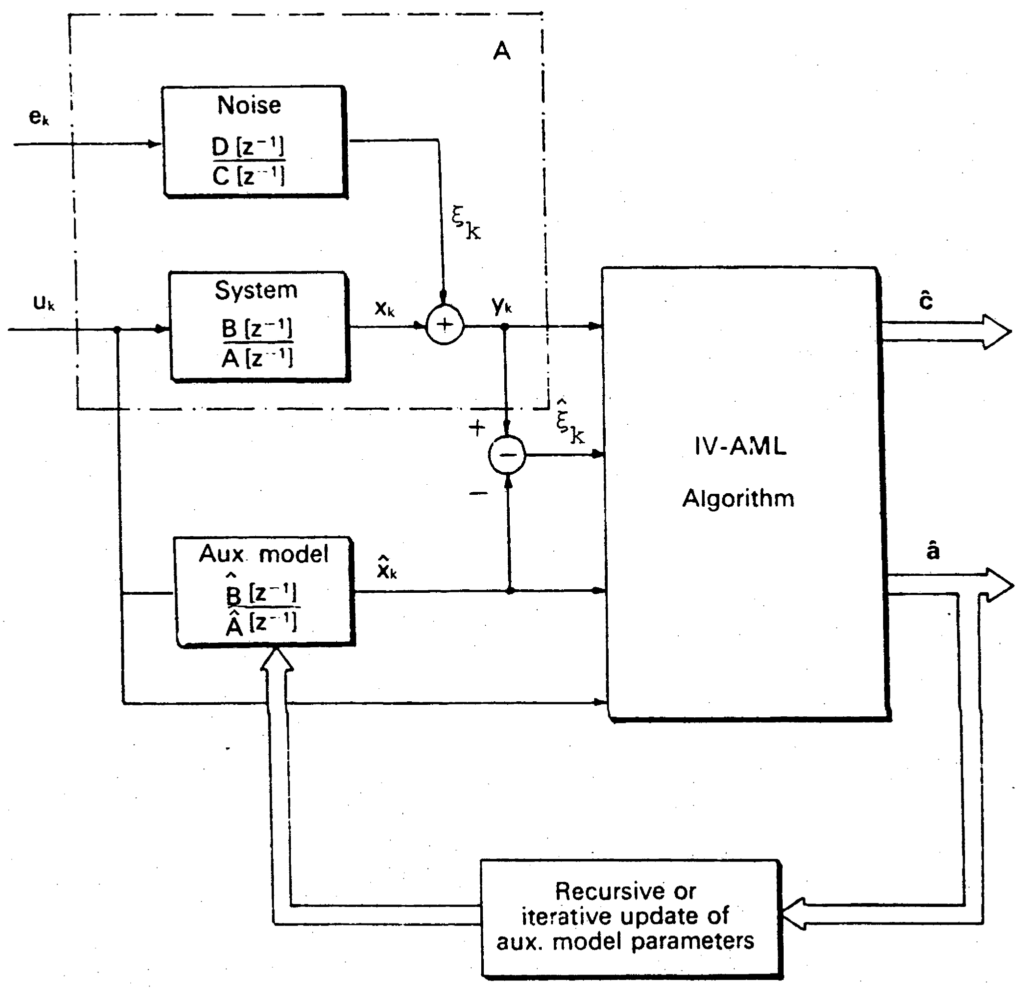
An iterative/recursive updating estimation procedure is used throughout in this study so that after a full recursive pass through the data (i.e., one iteration) according to equation (3.20), another is made until the parameter estimates converge. Within each iteration the auxiliary model outputs in equation (3.21), which are part of the IV vector $\hat{\underset{\sim}{x}}_k$, are generated using $\hat{\underset{\sim}{a}}$ estimates from the last recursive step of the previous iteration. For the first iteration, the RLS algorithm is used to obtain inititally biased estimates of $\underset{\sim}{a}$ for use in equation (3.21). A maximum of 6 to 8 iterative updatings seem to be required before the parameter estimates converge, often much less, say 2 to 3.

Subsequently, a model of the basic process based on these IV estimates is used to generate the final estimate $\hat{x}_k$ of the noise-

FIGURE 3.3

THE IV-AML APPROACH TO TIME SERIES

ANALYSIS OF TRANSFER FUNCTION MODELS



Source: Adapted from Young (1974), p.216.

free output x_k. This, in turn, yields an estimate $\hat{\xi}_k$ of the noise sequence ξ_k by reference to the equation:

$$\hat{\xi}_k = y_k - \hat{x}_k$$

In the second step of the estimation procedure, the sequence $\hat{\xi}_k$ obtained above, provides the input for the second recursive procedure, i.e., the AML algorithm. It provides consistent estimates of the noise model parameter vector $\underset{\sim}{c}$. By using the current estimates $\hat{\underset{\sim}{c}}$ of $\underset{\sim}{c}$, it also provides an estimate $\hat{e}_k$ of the white noise e_k by the relation:

$$\hat{e}_k = \hat{\xi}_k - \hat{\underset{\sim}{v}}_k^T \hat{\underset{\sim}{c}}$$

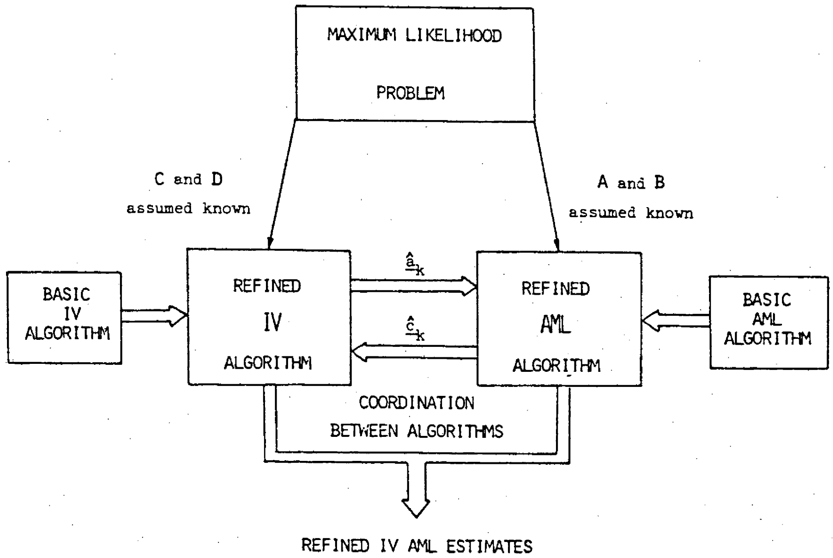
where $\hat{\underset{\sim}{v}}_k$ is the estimate of $\underset{\sim}{v}_k$ containing the lagged $\hat{\xi}_k$'s and $\hat{e}_k$'s. Theoretical and operational details like initial values for $\hat{\underset{\sim}{a}}_o$ and $\hat{P}_o$ are provided by Young (1972;1974) and Young and Jakeman (1979a).

Used together, the recursive IV and the recursive AML provide a complete recursive technique for consistent estimation of parameters in TF models. In fact, a more recent version of this technique, known as the 'refined' IV-AML algorithm, developed and reported by Young and Jakeman (1979a; 1980) and Jakeman and Young (1979), allows for coordinated estimation of the system and noise parameter vectors. It results in more efficient estimates.[1] This is achieved through a mechanism of communication between the two sub-algorithms as the solution proceeds as schematically represented in Figure 3.4. The IV-AML technique can also be unified within a maximum likelihood framework as discussed by Young (1976). This makes the approach statistically as rigorous as the conventional Box-Jenkins approach. In many respects, both of these approaches are comparable as shown in Table 3.1. However, the IV-AML technique

1 In fact they are asymptotically efficient which means that for very large sample sizes the estimates have as low a variance as is possible to obtain. An estimator is said to be asymptotically efficient if it is consistent and the variance of its limiting distribution is no larger than the variance of the limiting distribution of any other consistent estimator.

FIGURE 3.4

THE APPROACH TO EFFICIENT PARAMETER ESTIMATION IN 'REFINED IV-AML' ALGORITHM



Source: Young (1976, p.605).

has on-line potential and has been shown to provide greater insight into the system process as briefly discussed earlier in (i) and (ii) in Sub-Section 3.2.1. It is, therefore, considered at least complementary to the non-recursive approach (Young, 1974). Just as the *estimated* variance-covariance matrix emerged naturally from the RLS algorithm, so it does from IV-AML. For the basic and refined IV technique, it is given by:

$$\hat{P}^*_N = \hat{\sigma}^2 \hat{P}_N \tag{3.22}$$

where $\hat{\sigma}^2$ is the variance of the $\hat{e}_k$ sequence obtained from the corresponding AML algorithm.

TABLE 3.1

A COMPARISON OF TWO TIME SERIES ANALYSIS TECHNIQUES FOR ESTIMATING TRANSFER FUNCTION MODELS

Characteristics	Non-Recursive or Box-Jenkins Approach	Recursive Basic (Refined) IV-AML Approach
a) Basic TF model assumptions	Both the system and noise model outputs are generated by ARMA - like processes.	Not necessary to assume noise model output as generated by ARMA process (except for refined IV-AML).
b) General analytical approach	'Block' data analysis using maximum likelihood estimation.	Recursive analysis using instrumental variable modification to Ordinary Least Squares.
c) Noise model analysis	Simultaneously with the system model.	After the analysis of system model. (In refined IV-AML, separate but co-ordinated estimation).
d) Model structure identification	Highly interpretative; makes use of auto and cross-correlation coefficients.	Direct and systematic; the statistics from basic and refined IV algorithms are used to identify the structure of the system model.
e) Properties of parameter estimates	Consistent and asymptotically efficient.	Consistent and efficient enough for most practical purposes (refined IV-AML is consistent and asymptotically efficient).
f) Ability to update parameter estimates recursively	Possible only through a stage-wise process; also will require more computer storage.	Has inherent potential for updating parameter estimates and variance-covariance matrix.

Alternatively, should we not wish to use AML results in the system model structure identification stage (i.e., in finding values of m and n), then:

$$\hat{P}_N^{**} \approx \hat{\sigma}_\xi^2 \hat{P}_N \tag{3.23}$$

can be used where $\hat{\sigma}_\xi^2$ is the variance of the residuals $\hat{\xi}_k = y_k - \hat{x}_k$. This simplification avoids the identification and estimation of noise model parameters and has been shown to work extremely well in practice and in simulation tests (Young et al, 1980). In fact, it has proven as effective as the better estimate given by equation (3.22).

The IV-AML technique is available for use in a computer program package known as 'CAPTAIN'. However, as with other procedures, success in the use of CAPTAIN depends upon the TF complying with a few requirements which we present for completeness.

3.2.3 Theoretical Assumptions

For successful application of the recursive IV-AML technique, the TF model should necessarily satisfy a set of theoretical assumptions. They, as set out by Young et al (1971), include the following.

(i) The system process should be stable. That is, the roots of the characteristic equation $A(z^{-1}) = 0$ should be outside the unit circle in the complex plane.

(ii) The coefficients in $B(z^{-1})$ should not all be zero. Otherwise, of course, it means that the process is not activated by the input;

(iii) The polynomials $A(z^{-1})$ and $B(z^{-1})$ should have no common factors. This property is referred to as *observability*;

(iv) The input signal u_k should be persistently exciting; and

(v) The noise model should also satisfy the stability and observability properties; and should also be *minimum phase*, i.e. the roots of $D(z^{-1})$ should lie outside the unit circle in the complex plane.

In practice, these are implicitly assumed.

3.2.4 Computer Aided Program for Time series Analysis and Identification of Noisy systems (CAPTAIN) Package

The CAPTAIN package is built around several core programs, the most important of which implements the recursive IV-AML algorithms (Young and Jakeman, 1979b). The basic program for IV-AML identification and estimation is designed for the single input-single output TF model such as the rainfall-water storage model. Several enhancements are also available to this basic program. Among them are the TVAR facility for time varying parameter estimation, and the 'refined' IV-AML algorithm.

User manuals are now available in conversational mode FORTRAN (Mutch and Whitehead, 1975) as well as in command mode FORTRAN (Venn and Day, 1977). CAPTAIN allows the user to select various time series analysis options such as model order identification, parameter estimation, model simulation and validation. Also it provides the user with immediate visual output including graphical outputs on a visual display screen, such as the one used for this work, a Tectronix 4012 terminal with hard copying facilities. The visual-interactive operation of the package provides immediate information on the effect of the decisions of the analyst. Such a learning process is invaluable in time-series modelling. This facility is not available in other time series analysis programs such as the one of Box-Jenkins.[1]

Successful applications of CAPTAIN (or the IV-AML technique) to hydrologic systems have been described by Whitehead et al (1979) for rainfall run-off modelling of the Bedford-Ouse River in U.K. and Whitehead et al (1978) for run-off routing of the Murrumbidgee River in Australia. CAPTAIN has also been used in a hydrological context by Mackay et al (1980), Lyne (1979), Blunden and Moodie (1978) and Weeks (1977). But all the above applications involve run-off responses of catchments and relate also to the Australian situation. The present study attempts to apply CAPTAIN to rainfall - dam water storage in Sri Lanka. The 'basic' IV-AML is applied for model structure identification and preliminary estimation, and the 'refined' IV-AML is invoked for final estimation.

3.2.5 Model Statistics

As mentioned earlier, model structure identification in the CAPTAIN package can be invoked quite straightforwardly. For a range of given model orders, it computes statistics which emerge naturally from the IV technique as detailed by Young et al (1980).

---

1 The program for Box-Jenkins technique is marketed by ISCOL in U.K. The Natural Environment Research Council (NERC) in association with P.C.Young market the CAPTAIN package.

It may be recalled that the recursive IV algorithm generates an estimate of the variance-covariance matrix, P^* of the parameter estimation errors at each recursive step according to the relations (3.22) and (3.23). The first estimate is accurate for the refined IV-AML and the second is conservative for the basic IV-AML algorithm. If certain elements of P_k^* from either algorithm become large, it follows that the variance-covariance of the parameter estimation error has become large as might be expected if identifiability errors are encountered.

One way of monitoring change on this matrix is by computing the arithmetic mean of the diagonal elements of the final $\hat{P}_N^*$ or $\hat{P}_N^{**}$. It can, therefore, be interpreted as an overall estimation *Error Variance Norm* (EVN), i.e., for basic IV,

$$\text{EVN}(n,m) = \frac{1}{m+n+1} \sum_{i=1}^{m+n+1} \hat{p}_{ii}^{**}$$

where $\hat{p}_{ii}^{**}$ (i.e., $\hat{p}_{ij}^{**}$, $i=j$) is the i-th diagonal element of $\hat{P}_N^{**}$ corresponding to the estimated variance of the i-th parameter in $\hat{\underset{\sim}{a}}$.

Another more frequently used statistic is the *Normalised Error Variance Norm* (NEVN) given by

$$\text{NEVN}\ (n,m) = \frac{1}{m+n+1}\left(\sum_{i=1}^{n} \frac{\hat{p}_{ii}^{**}}{|\hat{a}_i|} + \sum_{i=0}^{m} \frac{\hat{p}_{n+i+1,n+i+1}^{**}}{|\hat{b}_i|}\right)$$

with $\hat{a}_i$ and $\hat{b}_i$ the final instrumental variable algorithm's estimates. The logarithm and percentage of EVN and NEVN are also quoted often. The percentage NEVN can be nicely interpreted as the average percentage parameter variance.

The statistic used to infer the explanatory power of the model is the 'R^2', which is defined as:

$$R^2 = 1 - \frac{\sum (y_i - \hat{x}_i)^2}{\sum y_i^2}, \qquad R^2 \leq 1$$

where $\hat{x}_i$ is the final IV estimate of the noise-free output given by equation (3.21).

In general, a low order model will adequately represent the system whenever the R^2 *tends* to the maximum for the full range of possible models and the NEVN is relatively low. Structure identification of the noise model is not so clear cut and the interested reader is referred to Box and Jenkins (1970) and Akaike (1974).

Of course, we have only outlined the general philosophy of the identification approach here and other checks must be carried out. The interested reader is referred to Young et al (1980) for a comprehensive strategy.

3.3 Stochastic Dynamic Water Supply Model

As we will see, a linear time-series model can, together with associated cascaded non-linearities provide a parametrically efficient description of the dynamic behaviour of dam water storage. But it does not explicitly incorporate the internal 'mechanics'; Firstly, because precise knowledge of the mechanisms (physical, chemical etc.) is not available; and secondly because there is no elaborate data on the system to verify any attempt at a detailed mechanistic description.[1]

1 However, it will be seen in Chapter 4 that for the water storage problem, the model is not completely a 'black-box' type since the 'linearisation' process takes into account a few key elements of hydrologic understanding of the system.

Rather the time series model assumes that a 'law of large systems' (young 1978; 1980) applies to the storage phenomenon so that a relatively simple mathematical description can explain the simple observable system behaviour. Thus the model is intended to describe only the dominant observable modes of the system. Such a broad description must incorporate a stochastic aspect as well in order to account for the associated effects of uncertainty. A stochastic component is also helpful to compensate for the effects of measurement error.

In this way, the TF model becomes statistically based. Quantification of the statistical uncertainty of the parameter estimates in the model provides a range over which each parameter in the model is known to vary to a given confidence (tolerance). As will be seen, Monte Carlo simulation is ideal for providing the associated uncertainty of the model output. In the process, water allocation policies can also be incorporated.

3.3.1 Stochastic - Monte Carlo Simulation

The term 'simulation' or 'computer simulation' is self-explanatory. In the context of mathematical models, Naylor et al (1966, p.3) define simulation as:

> 'a numerical technique for conducting experiments on a digital computer, which involves certain types of mathematical or logical models that describe the behaviour of a system over an extended period of time'.

Simulation experimentation is potentially very valuable for stochastic problems. When applied to problems having a probabilistic basis, it is referred to as stochastic or Monte Carlo simulation. For instance, it is relevant for a stochastic process which, in operation, is characterized by parameter estimates. Particularly, this is so when estimates take the form of probability distributions which are necessarily derived on the basis of statistical inference. Fundamentally then, Monte Carlo analysis entails the construction of a probabilistic model of the system to be studied. Subsequently, the system behaviour is simulated a large number of times with the model defined for each repetition. Each time, the values of the stochastic inputs and uncertain parameters are selected at random from their estimated parent probability distributions.

Whereas the selection of values of the stochastic input(s) is straightforward here, consideration of parameter uncertainty involves multivariate statistical procedures. The choice of the procedure is largely determined by the statistical properties of the estimates.

3.3.2 Relevant Statistical Properties of TF Model Parameter Estimates for Simulation

Statistical properties of en-bloc estimators of standard TF model parameters have been discussed by Pierce (1972). Extension of these theoretical results to recursive refined IV-AML estimation of TF models has been noted by Young and Jakeman (1979a). Two of their conclusions relevant to our purposes are that:

(i) $\underset{\sim}{\hat{a}}$ and $\underset{\sim}{\hat{c}}$, the estimates of the parameter vectors in the system and noise model, respectively, converge to the true parameter values $\underset{\sim}{a}$ and $\underset{\sim}{c}$ asymptotically and that the two sets are asymptotically independent; and,

(ii) the parameters will have limiting normal distributions and the sample variance-covariance matrix of the system model and noise model parameter estimates provide good estimates of their true variance-covariance matrices.

Further, it is also known that a multivariate normal distribution is defined for a vector of random variables where each element (or parameter) of the random vector is a random normal variable with given mean and variance. It follows then that the estimates of the system and noise model parameter vectors $\underset{\sim}{a}$ and $\underset{\sim}{c}$ originate from two independent multivariate normal distributions.

When considering a multivariate normal distribution where the elements (or parameters) therein are independent, the choice of random values for simulation is straightforward. If, however, the elements of the vector are not independent, which is generally the case, then their covariances need to be considered.

3.3.3 Monte Carlo Generation of TF Model Parameters

The vector of random normal variates for the system model, for instance, can be denoted by $\underset{\sim}{\alpha}$, which is of course (m+n+1) dimensional, and the mean vector by $\underset{\sim}{\hat{a}}$ such that:

$$E\{\underset{\sim}{\alpha}\} = \underset{\sim}{\hat{a}}$$

Also, its variance-covariance matrix can be represented by P^* where,

$$P^* = E\{(\underset{\sim}{\alpha}-\underset{\sim}{\hat{a}})(\underset{\sim}{\alpha}-\underset{\sim}{\hat{a}})^T\}$$

or

$$P^* = \begin{bmatrix} p_{11} & \cdots\cdots & p_{1,m+n+1} \\ \vdots & \ddots & \vdots \\ \vdots & p_{ij} & \vdots \\ \vdots & \ddots & \vdots \\ p_{m+n+1,1} & \cdots & p_{m+n+1,m+n+1} \end{bmatrix}$$

In this expression, p_{ij} (i=j) denotes the variance of the i-th element (or parameter), and p_{ij} ($i \neq j$) denotes the covariance between the i-th and j-th elements of the vector of random normal variates.

Conversely, the vector of random normal variates with a given mean vector $\hat{\underset{\sim}{a}}$ and variance-covariance matrix P^* can be generated making use of a theorem discussed by Anderson (1958, p.19) and Naylor et al (1966, pp.98-9). This can be restated as follows:

> 'If $\underset{\sim}{z}$ is a standard normal vector, i.e., it consists of independent normal variable components with zero mean and unit variance, there exists a unique lower triangular matrix L such that:
>
> $$\underset{\sim}{\alpha} = L\,\underset{\sim}{z} + \hat{\underset{\sim}{a}}$$
>
> and
>
> $$LL^T = P^* \qquad \text{(see footnote).'}$$

where

$$L = \begin{bmatrix} l_{11} & & & & 0 \\ \vdots & \ddots & & & \\ \vdots & & l_{ii} & & \\ \vdots & & & \ddots & \\ l_{m+n+1,1} & \cdots & & & l_{m+n+1,m+n+1} \end{bmatrix}$$

Operationally, the lower triangular decomposition of P^* can be performed in a series of steps as shown below:

$$l_{i1} = \frac{p_{i1}}{p_{11}^{\frac{1}{2}}} \qquad , 1 \leq i \leq (m+n+1)$$

$$l_{ii} = \left(p_{ii} - \sum_{k=1}^{i-1} p_{ik}^{\;2}\right)^{\frac{1}{2}} \qquad , 1 \leq i \leq (m+n+1)$$

Of course, P^* is a symmetric positive definite matrix and thus can be decomposed into a simpler lower triangular matrix such that:

$$LL^T = P^*.$$

$$l_{ij} = \frac{p_{ij} - \sum_{k=1}^{j-1} p_{ik}\, p_{jk}}{p_{ij}} \qquad , \; 1 < j < i < (m+n+1)$$

The procedure is relatively simple *especially* when the number of parameters in the vector $\underset{\sim}{a}$ is small. As an illustration let us consider a two parameter case, where

$$\underset{\sim}{\alpha} = \begin{bmatrix} \alpha_1 \\ \beta_0 \end{bmatrix} \quad \text{and} \quad \underset{\sim}{\hat{a}} = \begin{bmatrix} \hat{a}_1 \\ \hat{b}_0 \end{bmatrix}$$

And the variance-covariance matrix is P^* as before, where

$$P^* = \begin{bmatrix} p_1 & p_2 \\ p_2 & p_4 \end{bmatrix} \tag{3.24}$$

Again making use of the theorem,

$$\underset{\sim}{\alpha} = L\,\underset{\sim}{z} + \underset{\sim}{\hat{a}}$$

or

$$\begin{bmatrix} \alpha_1 \\ \beta_0 \end{bmatrix} = \begin{bmatrix} l_1 & 0 \\ l_2 & l_4 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} \hat{a}_1 \\ \hat{b}_0 \end{bmatrix} \tag{3.25}$$

It could also be shown that

$$P^* = \begin{bmatrix} l_1^2 & l_1 l_2 \\ l_1 l_2 & l_2^2 + l_4^2 \end{bmatrix} \tag{3.26}$$

since $LL^T = P^*$.

From equations (3.24) and (3.26) above,

$$l_1 = p_1^{\frac{1}{2}}$$

$$l_2 = \frac{p_2}{p_1^{\frac{1}{2}}}$$

$$l_4 = \sqrt{\left(p_4 - \frac{p_2^2}{p_1}\right)}$$

By substituting these results into equation (3.25), we have

$$\underset{\sim}{\alpha} = \begin{bmatrix} \alpha_1 \\ \beta_0 \end{bmatrix} = \begin{bmatrix} p_1^{\frac{1}{2}} \; z_1 \\ \dfrac{p_2}{p_1^{\frac{1}{2}}} \cdot z_1 + \sqrt{\left(p_4 + \dfrac{p_2^2}{p_1}\right)} \; z_2 \end{bmatrix} + \begin{bmatrix} \hat{a}_1 \\ \hat{b}_0 \end{bmatrix} \tag{3.27}$$

From equation (3.27), it is clear that α_1 and β_0 are generated as correlated random normal variates for the parameters a_1 and b_0 by making use of the sample variance-covariance matrix of the latter and two independent standard normal variates, z_1 and z_2.

Literature on standard normal variates and their application in Monte Carlo analysis is vast. For example, Hamersley and Handscomb (1964) and Handscomb (1969) have elaborated the theoretical aspects. Practical details of the generation of standard normal variates for use in Monte Carlo analysis have been discussed, for example, by Moy (1969) and Naylor et al (1966). Today it is one of the standard statistical facilities in computer software and is easy to implement.

From the foregoing it is clear that the parameter estimates of $\underset{\sim}{a}$ and the covariance matrix P^* constitute the essential inputs in the development of a probabilistic model for the system process. In a similar fashion, $\underset{\sim}{\hat{c}}$ and its variance-covariance matrix can be used for defining the random normal variates for the noise model. Random normal variates so generated for the system model as well as the noise model allow a probabilistic description of the water storage process. This, in turn, constitutes the basis for Monte Carlo simulation.

3.3.4 Simulation Output

It is now clear that the inherent probabilistic nature of the TF model allows Monte Carlo simulation. By performing such simulation, storage levels can be forecast in a probabilistic sense. In contrast to deterministic forecasts, the stochastic element allows for forecasts not only in terms of a mean storage but also of the probability distribution around this mean. The approach has been successfully adopted before, for example, by Whitehead and Young (1979) in the context of a study of water quality in river systems.

By repeating the simulation a number of times, the probability density function of storage y_k at each time step, k, can be built-up. Of course, the more simulation runs the more accurate is the probability density, since the deviation of the realized probability density function from the true one is obviously a function of the number of simulations. A Kolmogorov-Renyi statistic discussed by Spear (1970) can be invoked which says, for example, that 186 (2055) simulations are required to obtain the cumulative distribution of y_k accurate to 0.10 (0.03). It may be noted that raising the accuracy from 0.10 to 0.03 involves eleven times as much computer time.

The mean and standard deviation computed for each time step can be extended to describe the overall storage behaviour with a mean and confidence band. This approach, previously used by Whitehead and Young (1979), is adopted in the present study.

However, a stochastic water storage model is incomplete unless it gives adequate consideration to the stochasticity of the input, which is in this case the rainfall.

3.3.5 Considerations of Rainfall Stochasticity

Rainfall in the Dry Zone is noted for its variability both in terms of quantity and distribution. One possible approach to incorporate this element has been mentioned earlier. It makes use of the statistical distribution of rainfall at each time step. But an appropriate description of rainfall is not available for this study. Besides, apart from the limitation of time, the number of years of rainfall data is felt inadequate to undertake a worthwhile statistical analysis of rainfall.

An alternative approach adopted here is to carry out simulations of storage for a range of years. Twenty consecutive years' rainfall data[1] are available for this study. Simulation models of storage for different years of rainfall data can provide insight into the likely variability in storage behaviour.

3.3.6 Incorporation and Evaluation of Irrigation Strategies

In the simulation model of storage for a given rainfall year, a sequence of withdrawals of water based upon recent rainfall levels and the rice crop's water requirements can be incorporated for each simulation and the behaviour of the water storage levels observed. This incorporation only involves a possible subtraction at each time step of a policy-calculated withdrawal from the TF model output. These observations and certain other criteria can be used to evaluate different withdrawal strategies to irrigate the dry season's rice crop. Two of the criteria mentioned in the previous chapter are the total amount of irrigation applied and the storage remaining at the end of the year. With these the variability of

1 Data source is considered in Appendix A.

the year-end storage (e.g., the standard deviation) and the probability of emptying the dam are also included.

It will be seen that implicit in the withdrawal strategy options is the timing of the first irrigation and sowing of the second (or dry) season's rice which are one to two weeks apart. Details of the strategies in relation to the actual dam situation, however, are postponed until Chapter 5 after the development of the simulation model of dam water storage in the next chapter.

CHAPTER 4

MODELS FOR WATER STORAGE:
RESULTS AND DISCUSSION

This chapter is devoted to the actual identification and estimation of a time series model from data obtained for a specific dam site. This is then used to perform stochastic simulation of water storage (or supply). In Section 4.1, a choice is made of the sampling interval for the analysis of our system, and a preliminary model structure is also determined. Section 4.2 discusses the sources of non-linearity in the process and describes and illustrates the effects of a procedure for accounting for this in the time series model. This is based on a non-linear modification of the input (raw rainfall) to yield a new input which we call *effective rainfall*, u_k^*; in this way, the TF model between u_k^* and the storage, y_k remains linear in the parameters.

In Section 4.3, the results of the identification and estimation of the TF model for storage are presented. This includes both basic and refined IV-AML results and some implications in terms of the identified model's physical interpretation and general applicability. The stochastic simulation results are presented in Section 4.4. These are used to highlight the probabilistic nature of the model and water storage extremes. Also presented in this section are the model's probability density function forecasts of all storage levels at the two periods of major interest, the beginning and the end of the dry season. The overall results are summarised in Section 4.5.

In Chapter 3, we discussed in detail the identification and estimation procedure for the transfer function (TF) model of a dynamic system. Yet, it was based on two important premises: namely, that the system is linear and that an appropriate time interval is chosen for the sampling of the rainfall input and the water storage output. However, as noted earlier, the rainfall-water storage system is non-linear. In addition, the input-output data used were collected well before the analysis of this dissertation was commenced so that an optimum sampling interval could not be predetermined. We now discuss these problems.

4.1 Rainfall-Water Storage Process: Sampling of Data and Preliminary Model Structure

The sampled data on rainfall and water storage level in the dam for the present study are available for a period of slightly less than a year, from October 1976 to August 1977; and the actual sampling interval was a week (Mahendrarajah, 1978, p.28). This allows the analysis of the data either on a weekly basis or on the basis of multiples of weeks. Fortunately, weekly samples seem to sufficiently expose the dynamic relationship between rainfall and storage and anything coarser than weekly would not provide a reasonable number of sampling points. The sample number of 45 weeks for storage[1] turned out to be adequate for model identification and estimation but, as we shall see, not adequate enough to tune the non-linear aspects to our total satisfaction. The rainfall data are, in fact, available for a slightly longer period, i.e. 48 weeks (see Figure A.1 and Table A.1 in Appendix A).

1 It may not be clear to the reader as to how natural water storage levels are obtained while, in actual fact, the dam water is in continuous use for irrigation. Storage levels without withdrawals for our analysis are obtained by adding on the amounts withdrawn to the level that remains.

The use of the CAPTAIN package to analyse these data is straightforward when the system is linear. However, being non-linear, the rainfall-storage data requires an initial process of linearisation before parameterization. Yet, in order to determine the type of linearisation, CAPTAIN can be invoked. Model structure identification proceeds as it would for input-output data that are linearly related. Thus, a set of different but plausible model structures are considered as they are shown in Exhibit B.1 of Appendix B. Recall that the choice of a suitable model structure is based on changes in the NEVN and R^2 statistics. An examination of the statistics therein reveals that:

$$y_k = \frac{b_o}{1+a_1 z^{-1}} u_k + \xi_k$$

or

$$\left.\begin{aligned} x_k &= - a_1 x_{k-1} + b_o u_k \\ y_k &= x_k + \xi_k \end{aligned}\right\} \qquad (4.1)$$

is the most appropriate model form, since it is associated with the lowest NEVN and relatively high R^2, although the latter, predictably, has a low absolute value ($R^2 = 0.50$). Also the model is simple and intuitively attractive.

This is because it describes the level of storage in just two parameters. In fact, equation (4.1) says that the storage level at week k is a combined effect of the rainfall during that week and the storage level observed in the previous (k-1) week. However, in its current form, its ability to describe the data is rather poor as reflected by the low R^2. This is due to the non-linearity in the impulse response for storage. By identification of the sources of non-linearity and appropriate compensation the explanatory power of the model can be improved.

4.2 Linearisation of the Data and Effective Rainfall Series

Non-linearity in rainfall-storage data appears to be the consequence of the many interceptions and diversions of actual rainfall in the catchment before it collects in the dam.

4.2.1 Sources of Non-Linearity

Only a portion of the rainfall leads to a response or change in storage in the dam. A considerable portion is lost to the soil in the catchment as well as to the atmosphere via evapotranspiration. Evapotranspiration, which is temperature dependent, constitutes the main element of atmospheric loss; whereas, the losses to the soil take place via deep percolation and seepage. Such losses are largely a function of soil moisture 'level'. Saturated soils tend to leave a larger proportion of incident rain in run-off which, in turn, can lead to increments in storage in the dam. On the other hand, a low rainfall in a dry soil can all be lost especially in hot weather. Such considerations lead to the development of an *effective* rainfall measure: in simple terms, this reflects an effective portion of the incident rainfall in the catchment which results in storage change.

The effective rainfall measure allows for such factors as evapotranspiration and soil moisture; and it represents an estimate of the *true* input to which the storage responds. Furthermore, previous research (Whitehead and Young, 1975) has shown that the remaining response is primarily linear in form; in other words, most non-linearity in the response can be eliminated by the effective rainfall compensation. The modification process of obtaining effective rainfall from the basic or actual rainfall is referred to here as *prefiltering*.

4.2.2 Modification of Rainfall for Soil Moisture Effects

In the present study, two *filters* are employed to obtain an effective rainfall sequence from the actual rainfall. The procedure is similar to the one adopted by Whitehead and Young (1979) and Mackay et al (1980). It involves pre-filtering the raw rainfall for:

(i) soil moisture; and

(ii) temperature.

The modifications to actual rainfall to compensate for soil moisture levels involve accounting for antecedent precipitation by means of a simple and parametrically efficient exponential weighting of the rainfall into the past. A measure of soil moisture content or 'level' is obtained by filtering the rainfall, u_k, by means of a discrete first order filter of the form:

$$s_k = s_{k-1} + \frac{1}{T_c}(u_k - s_{k-1}) \qquad (4.2)$$

This represents an exponential smoothing into the past[1] so that the soil moisture value, s_k, is larger if the rainfall u_k has recently been continual, for example, than if it had not. The time constant T_c is associated with the soil moisture dynamics and determines how far into the past the exponential weighting is important. The new effective rainfall series, u_k^* is generated such that:

$$u_k^* = \left(\frac{s_k}{s_{max}}\right)^p \cdot u_k \quad \text{where } s_{max} = \max_k \{s_k\} \qquad (4.3)$$

The quantity (s_k/s_{max}) in equation (4.3) is the fractional weighting given to the rainfall. Thus, the weighting is high if it has been raining consistently over the period determined by the time constant

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1 It can also be considered as a model of the dynamics associated with soil moisture changes; i.e.the soil moisture lag process.

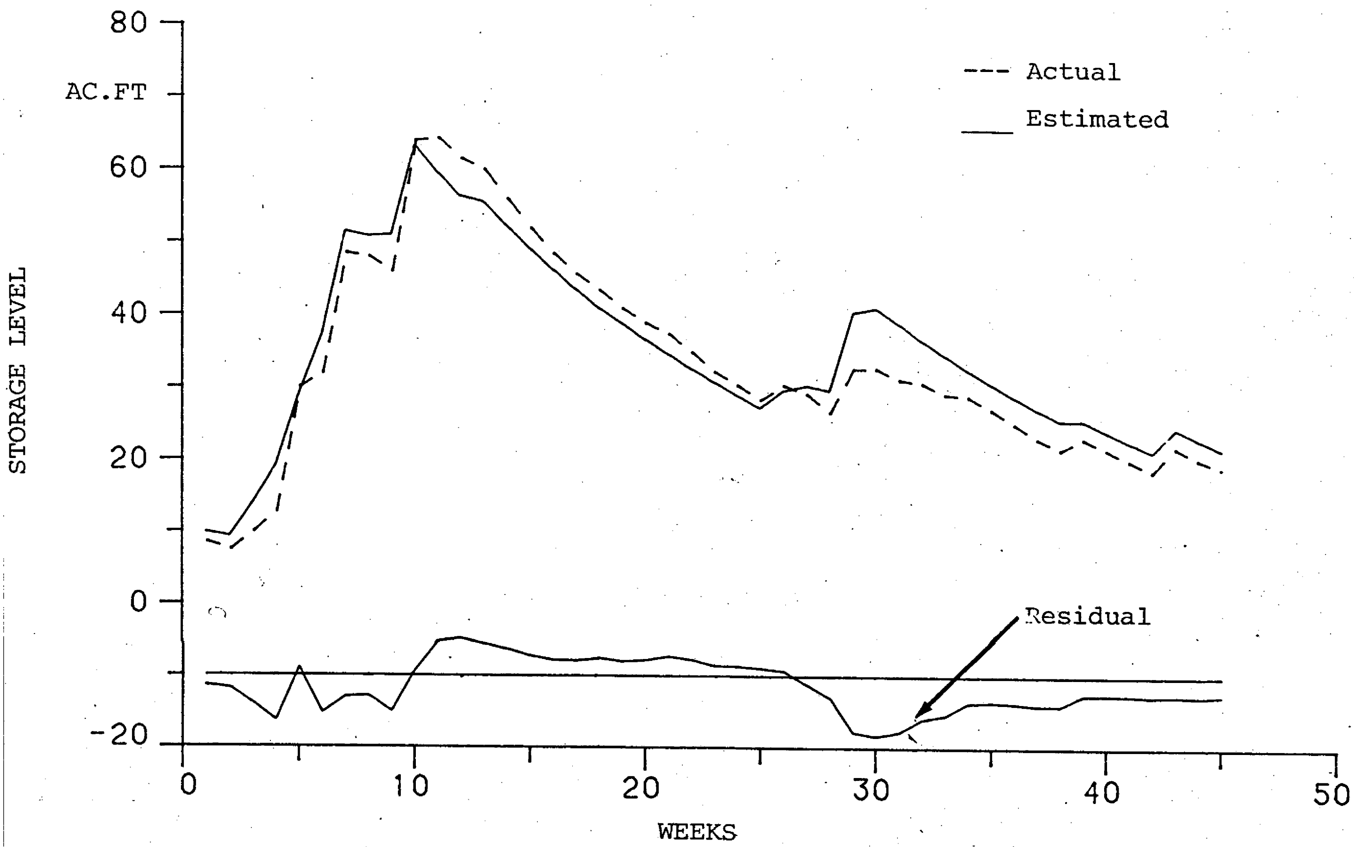
T_c, and most rainfall u_k then appears as effective rainfall with little loss to the already moist soil. In past studies, the value of p chosen has usually been unity; but recently in Mackay et al (1980), it was found that values of p when greater than unity (i.e. p>1) increased effective rainfall near peaks relative to the periods when the rainfall is low.

This added feature has been used advantageously in the present problem to attach differential weighting to the wet and the dry season rainfalls. It may be recalled that the rainfall during the wet season, apart from being relatively high, is fairly even in distribution in comparison to the dry season's rainfall. The actual values of T_c and p in equations (4.2) and (4.3) for the present problem are 6 weeks and 1.5 respectively. These are chosen by trial and error. An alternative would have been to 'optimise' the choice of these coefficients by some automatic hill climbing procedure but this was thought unnecessary in the present study. In practice, the soil moisture filter is well behaved in that small changes in T_c yield only small changes in the effective rainfall and it has been observed that there is only one optimum value of T_c which best linearises the relationship between effective rainfall and storage. In this way, it is easy to home in adequately closely to the appropriate value of T_c.

Next, equation (4.1) was estimated using the modified rainfall sequence given by (4.2) and (4.3) instead of the actual rainfall and the resultant model output and output are compared in Figure 4.1. The remaining deficiency of fit of the model was examined employing the TVAR facility of CAPTAIN. As mentioned earlier in Section 3.2 of Chapter 3, the TVAR Subprogram, when

FIGURE 4.1

ACTUAL WATER STORAGE AND BASIC IV-AML MODEL
WITHOUT TEMPERATURE INCORPORATION ON INPUT

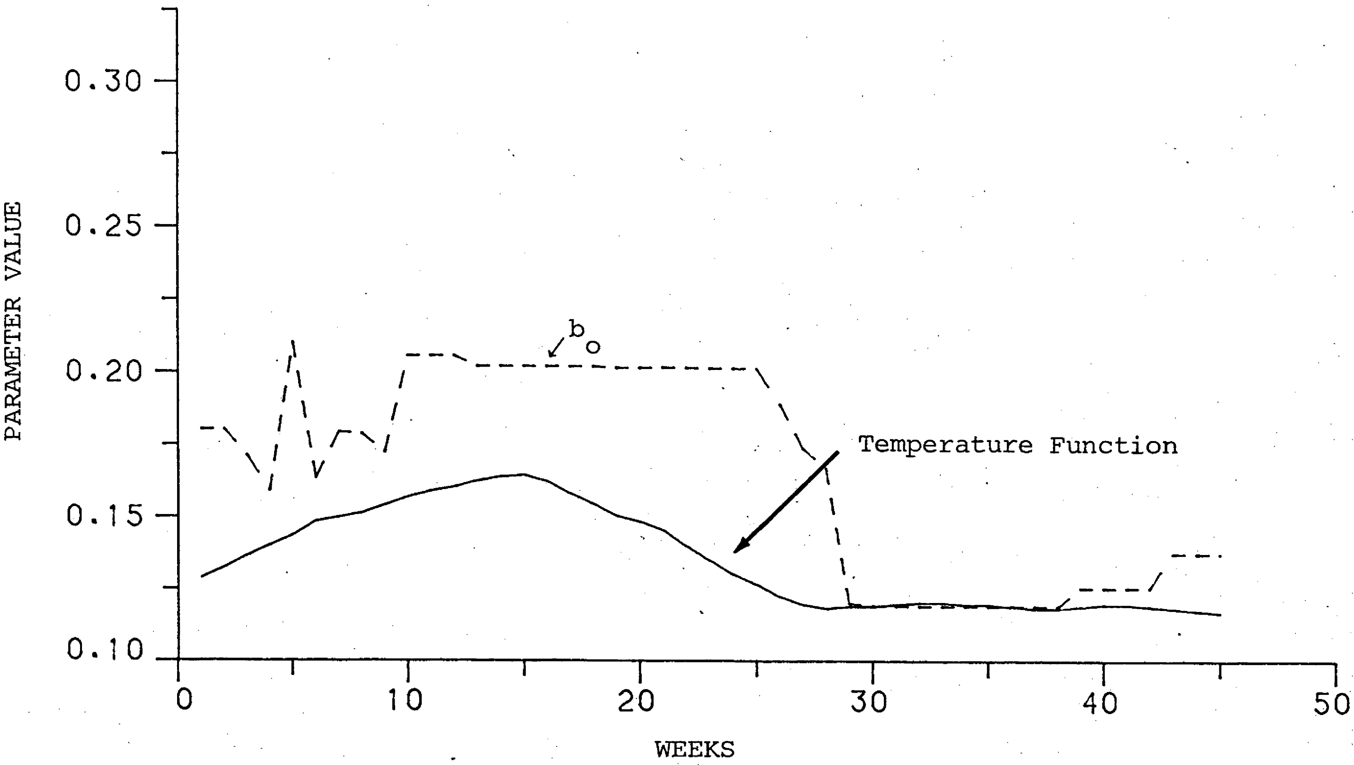


invoked, provides insight into the variability of a parameter value over time. Allowing the b_o parameter in the model to be time-varying[1] reveals the lack of constancy of b_o over time as shown in Figure 4.2. It is interesting to note that, as expected, the actual variation obtained for the b_o parameter has similarities to the pattern of variation of weather, *especially* the ambient temperature.

1 In CAPTAIN, this is according to a simple *random walk* which provides a flexible class of possible parameter variability with low parameterization. See Young and Jakeman (1980).

FIGURE 4.2

VARIATION OF b_O PARAMETER AGAINST
A TEMPERATURE FUNCTION (4.4)



High b_o values occur during the wet season and up to mid-May covering the rainfall period during the dry season. For the driest months from June to the end of August, significantly lower b_o values are manifested. Although, obviously this is the combined consequence of many climatic factors, the variation in temperature appears to be the major determinant. This suggests the need to consider a temperature function as well in *pre-filtering* in order to obtain a better measure of effective rainfall.

4.2.3 Compensation for the Effects of Temperature

Low temperatures reduce the evapotranspiration and hence tend to increase the effective rainfall compared to periods when the temperature is high. Such effects, if not adequately accounted for, lead to overestimation of storage during dry spells and underestimation during wet periods. This seems to be the case in the present problem, as evident from Figure 4.1. In order to compensate for such temperature induced evapotranspiration effects, a temperature function is incorporated in the *pre-filtering* process. It modulates the actual rainfall, u_k by a factor proportional to the difference between the prevailing mean weekly temperature, T_k and a notional maximum temperature, T_m, such that:

$$u_k' = \phi(T_m - T_k)\ u_k \tag{4.4}$$

where, ϕ is a proportionality constant, 0.014 in value in the present study; and T_m is chosen to be 100 degrees Farenheit. This temperature function is compared with the b_o parameter variation in Figure 4.2. Clearly, the temperature function does not seem to explain all the variation of b_o. Even the same function with a modification to raise $(T_m - T_k)$ to a power, say q (where $q>1$), may be more desirable. However, for our purposes the function of the form of equation (4.4) has proven to be adequate. In fact, this temperature compensation approach is similar to the one used by Whitehead and Young (1975) and Whitehead et al (1979).

The temperature compensated rainfall sequence, u_k' serves as the input for the soil moisture filtering process outlined earlier with equations (4.2) and (4.3). The overall *pre-filtering* process leading to the ultimate effective or filtered weekly rainfall series, u_k^* can be summarised as follows:

$$\left.\begin{aligned} u_k' &= 0.014\,(100-T_k)\,u_k \\ s_k &= s_{k-1} + \frac{1}{6}\,(u_k' - s_{k-1}) \\ u_k^* &= \left(\frac{s_k}{s_{max}}\right)^{1.5} \cdot u_k' \end{aligned}\right\} \qquad (4.5)$$

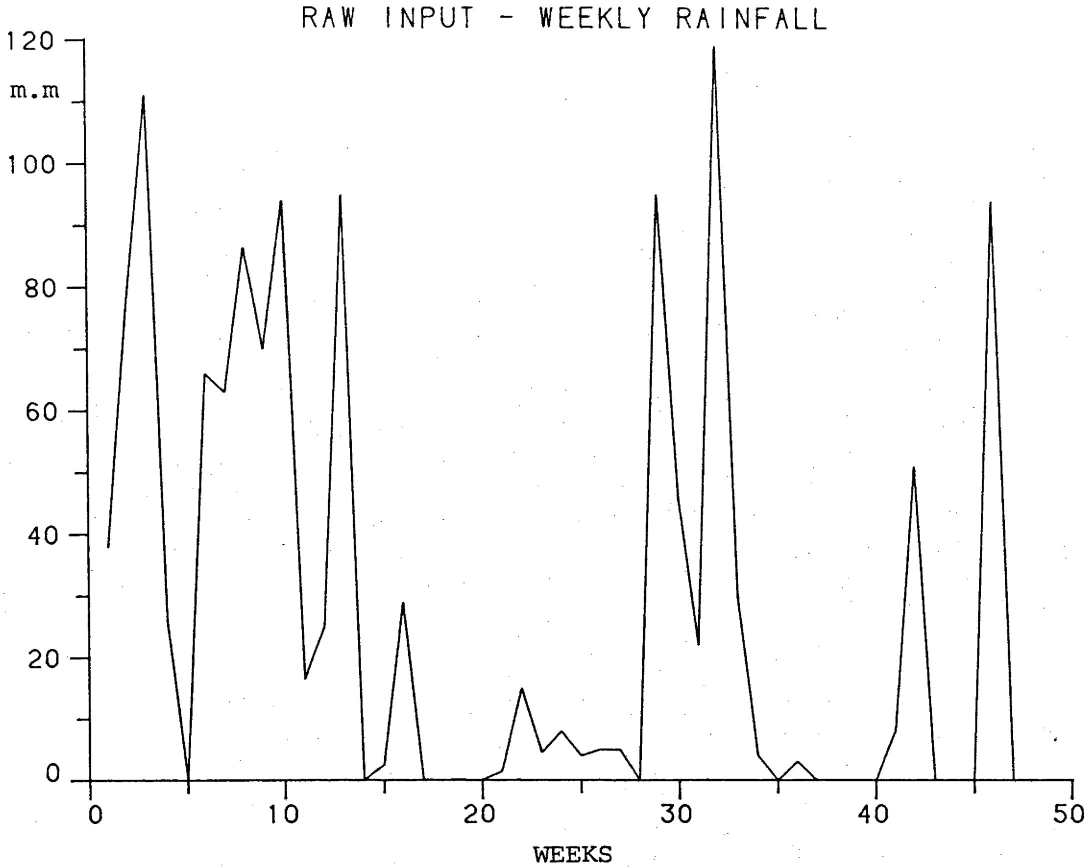
where $s_{max} = \max_k \{s_k\}$ and $k=1,\ldots,45$.

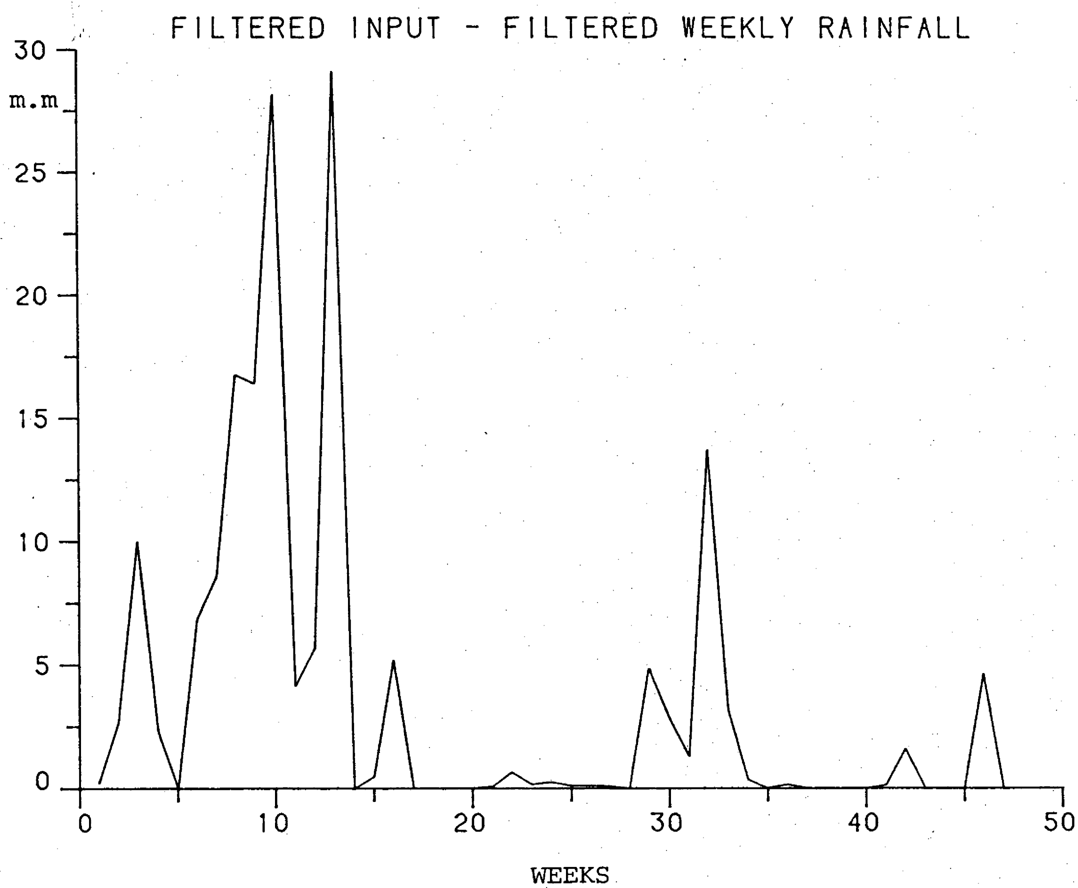
The effect of the filtering process on actual or 'raw' rainfall is shown in Figure 4.3. It may be noticed how low the effective rainfall is from weeks 20 to 28 (i.e. during February and March) and from 40 to 48 (i.e. the months of July and August) relative to the actual rainfall. These demonstrate the workings of the soil moisture filter when it has not been raining. In general, it may also be noted that actual rainfall in the second half of the year produces relatively less effective rainfall due to the temperature modification represented by equation (4.4). Another important feature which can be seen from this figure is that the effective or filtered rainfall is not defined directly as the 'rainfall excess' since it has not been attempted to equate the volume of effective rainfall to that of storage increment. Finally, we note that such an effective rainfall measure bears some similarity to the *antecedent precipitation index* (see Weyman, 1975) used in conventional hydrology.

FIGURE 4.3

ACTUAL AND EFFECTIVE WEEKLY

RAINFALL SERIES, 1976/77





The estimation of the model of the form of equation (4.1) now with the effective rainfall as input and storage as output provides a good fit to the storage data. The model output and the observed noisy output (in dashed line) are graphed in Figure 4.4 along with the residuals. Also, it will be seen that this very same first order model is, in fact, the best instrumental variable (IV) model for the effective rainfall sequence given by (4.5).

4.3 IV-AML Estimation of TF Model for Storage

With the basic non-linearities eliminated by *pre-filtering*, the effective rainfall-water storage data are now approximately linearly related.[1] Thus the usual procedure of model structure identification can be performed employing the CAPTAIN package.

4.3.1 Model Structure Identification

Using the effective input series defined by equation (4.5), the linear model structure

$$y_k = \frac{B(z^{-1})}{A(z^{-1})} \; u_k^* + \xi_k \tag{4.6}$$

can now be invoked as the relevant representation. Identification of this model with the effective rainfall as input and the observed storage level as output yields the results summarised in Table 4.1. Clearly, the best linear model is one with order (n,m) = (1,0). This is because it has the lowest average parameter variance (NEVN) at ·12 per cent and, with an R^2 of 0.979, it explains the output data at least as well as any of the other models.

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1 This assumption is based on the very much improved model explanation of the storage data and not on any rigorous statistical checks.

FIGURE 4.4

ACTUAL WATER STORAGE AND BASIC IV-AML MODEL

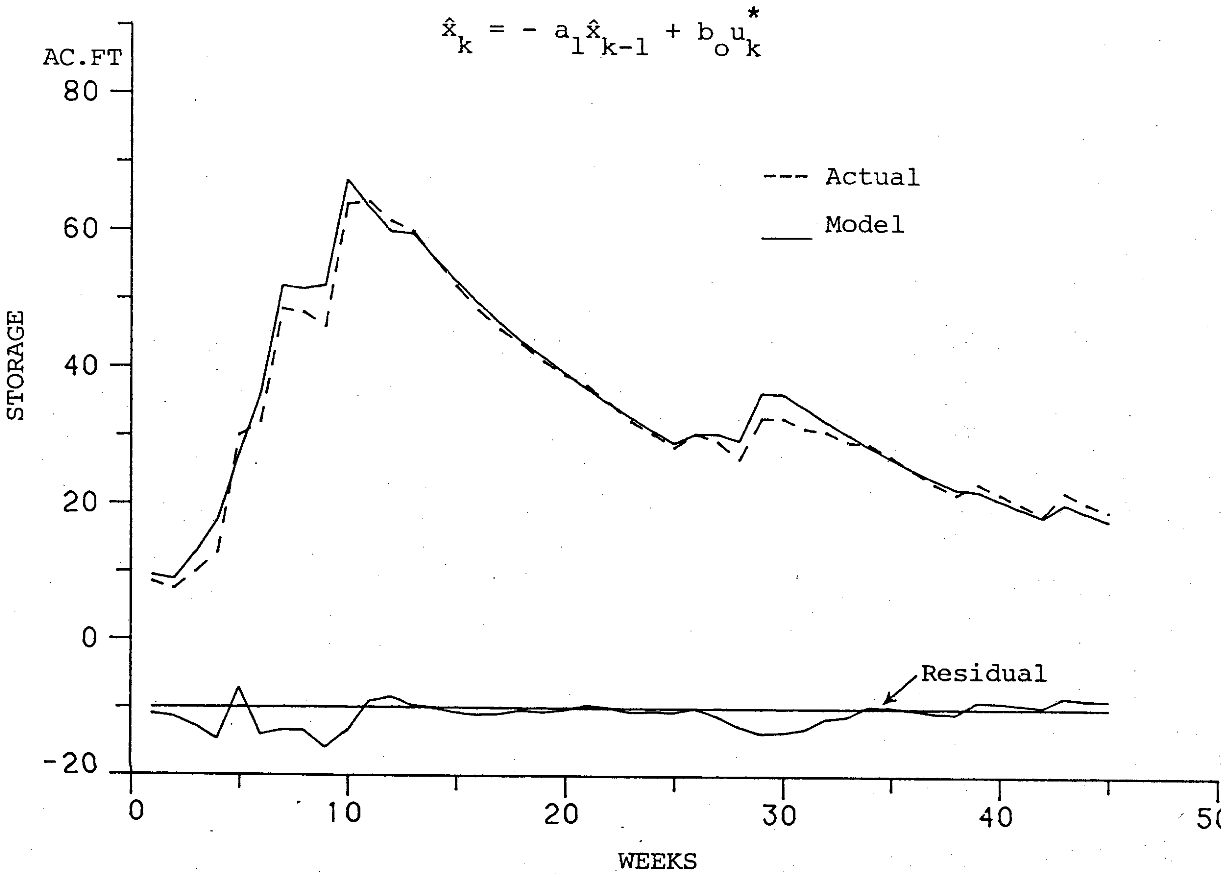


TABLE 4.1

MODEL ORDERS AND RELATED STATISTICS FOR EFFECTIVE RAINFALL-WATER STORAGE TRANSFER FUNCTION

Model Order (n,m)	R^2	Percent NEVN
(1,0)	0.979	0.12
(1,1)	0.969	5.71
(1,2)	0.935	3.53
(2,0)	0.967	10.41
(2,1)	unstable model	
(3,0)	0.971	2.52
(3,1)	unstable model	

Note: summarised from Exhibit B.2 of Appendix B.

It should be clear that Figure 4.4 represents the instrumental variable (IV) estimate of the model output $\hat{x}_k$, where

$$\hat{x}_k = - a_1\hat{x}_{k-1} + b_o u_k^*$$

The approximate maximum likelihood (AML) algorithm was implemented to model the residual sequence, $\hat{\xi}_k = y_k - \hat{x}_k$. Omitting the details, the structure identified for this model is purely auto-regressive (AR) of first order, viz.

$$\hat{\xi}_k = - c_1\hat{\xi}_{k-1} + \hat{e}_k$$

or

$$\hat{\xi}_k = \frac{1}{1+c_1 z^{-1}} \hat{e}_k$$

4.3.2 Model Estimation: Basic IV-AML and Refined IV-AML Results

The interactive procedure of recursive IV-AML estimation for the system model and noise model identified above is displayed in Appendix B. The estimation results are presented in Table 4.2. This also presents the results of the 'refined' recursive IV-AML estimation of the system and noise models. The main purpose of this estimation is to obtain accurate and low variance estimates for use in Monte Carlo simulation. The model fit obtained and the actual noisy storage output are graphed in Figure 4.5 along with the residual which is just the difference between the two. A close comparison of the model fit in this figure with Figure 4.5 for the basic IV-AML model reveals a slightly better overall fit in refined IV-AML estimation. Unfortunately, the refined algorithm used in

TABLE 4.2

ESTIMATES OF TRANSFER FUNCTION MODEL PARAMETERS AND RELATED STATISTICS

Parameter/ Statistic	'Basic' IV-AML	'Refined' IV-AML
a_1	-0.940 (0.0083)	-0.938 (0.0028)
b_o	0.640 (0.0398)	0.687 (0.0220)
Cov (a_1, b_o)	n.a	4.98×10^{-5}
c_1	0.560 (0.1249)	0.587 (0.0853)
$\hat{\sigma}^2$	3.508	3.871
R^2	0.979	0.972
Log EVN	-7.100	-8.311
Log NEVN	-6.693	-7.940

Notes: (a) Figures in parenthesis represent the standard errors of estimates.

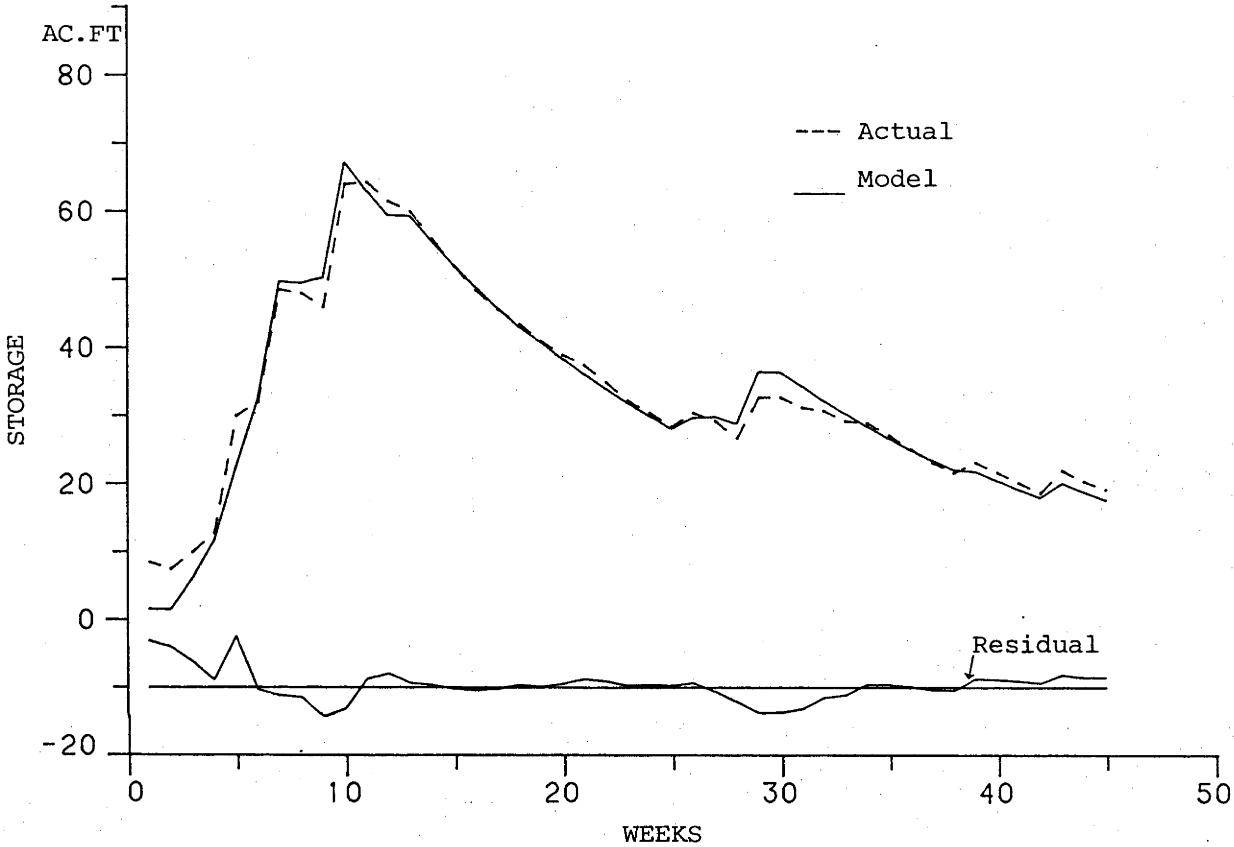
(b) n.a - not available explicitly.

this study did not at the time have incorporated the facility to set an initial non-zero output level. This is reflected in a poor fit to the first few weeks of data and an artificial low R^2.

A closer examination of Table 4.2 reveals that both the basic and refined IV-AML estimations have the ability to explain around 97 per cent of the variability in water storage data. But, as expected, the refined IV-AML model is associated with relatively lower EVN and NEVN statistics. The standard deviations of the parameter estimates are very much lower in this model than those of the basic IV-AML model.

FIGURE 4.5

ACTUAL WATER STORAGE AND REFINED IV-AML MODEL



The refined IV-AML results are quoted for future reference, viz.

$$y_k = \frac{0.687}{1-0.938z^{-1}} \; u_k^* + \frac{1}{1-0.587z^{-1}} \; \hat{e}_k$$

Or equivalently,

$$\left.\begin{aligned} \hat{x}_k &= 0.938\ \hat{x}_{k-1} + 0.687\ u_k^* \\ \hat{\xi}_k &= 0.587\ \hat{\xi}_{k-1} + \hat{e}_k \\ y_k &= \hat{x}_k + \hat{\xi}_k \end{aligned}\right\} \qquad (4.7)$$

The variance of the white noise sequence $\hat{e}_k$ is estimated at 3.871. It must also be noted that the system and noise models are themselves inherently stochastic, each having a variance-covariance matrix associated with the parameters in them.

4.3.3 Some Implications

Obviously, the model is simple and parametrically efficient; the system process is defined in two parameters, while the noise component is explained by only a single parameter. As noted earlier, for the storage phenomena, the system model component offers an interpretation which is intuitively appealing and relates the storage at a particular time to that at the previous time and the effective rainfall. Based on estimates, the storage in acre feet in the current week is equal to 93.8 per cent of the previous week's storage plus 68.7 per cent of the current week's effective rainfall in millimetres.[1] Apart from suggesting the recursive nature of the water storage dynamics, this has other important implications.

In forecasting storage, for instance, the procedure takes into account not simply the total rainfall, but also the distribution over the preceding weeks. This is made possible via:

(a) the effective rainfall computation which weights the actual rainfall measurement into the past, thereby introducing into the model a 'memory' element of a length determined by the time constant T_c; and

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1 It is not attempted to convert the units of measurement completely to one system. The analysis is pursued with the same units as in the data source: rainfall in Metric units (mm) and storage in the Imperial system (Ac.ft).

(b) the TF model form itself which, in this first order case, also weights the effective rainfall exponentially into the past with a time constant defined in terms of the value of $\hat{a}_1$.

Another important point that emerges from the mechanics of modelling relates to the extension of the model to other small dams. Clearly, the features of the catchment of the dam are reflected in the dynamics of the system; and they are manifested in the nature of the model and the values of the parameter estimates. Given the fact that each catchment is unique in a hydrologic context, each dam requires its own storage TF model. Moreover, even for the same dam, significant changes in the catchment (for example, deforestation) might call for a re-estimation of the model. Besides, the noted variability in rainfall itself may change parameter values from year to year. This emphasises the need for validation of the previously estimated model with several more years' rainfall and storage data.

The task of estimation and validation of individual models for dams would not appear to be an insurmountable one, *especially* when one considers the limited requirements in respect of data. Apart from the inherent potential to effectively make use of noisy data from crude measures, the approach depends upon a bare minimum of information. Rainfall measures and storage levels are only required on a weekly basis. The only other requirement is the weekly average temperature which is not only relatively constant for a considerable geographical area but also relatively stable over years for the same region. District or regional temperature records from such sources as Meteorological Stations can be used to substitute for on-site measures. Clearly, the advantages that stem from less demanding data requirements add a further dimension to the computational merits discussed in the previous chapter.

It is felt that the importance of such a water storage model cannot be overemphasised especially when one considers its value in achieving the ultimate purpose; namely optimisation of the water resource. This is discussed in the next section, which describes a stochastic simulation water management model for which the primary requirements are the storage TF model estimates obtained in this section. However, the estimates and the ensuing models based upon them suffer from a basic limitation: the estimates are based on only a *single year's rainfall and storage data*. As a result, the model at this point in time is limited in scope and, therefore, *the conclusions have to be treated with caution until further data is acquired and used to validate the present preliminary model.*

4.4 Stochastic-Monte Carlo Simulation Model for Storage

The ultimate aim of stochastic simulation in the present research is to evolve an analytical framework for the allocation of water resources of the small dam. A supplementary objective, however, is to make probabilistic forecasts and gain insight into the variability of the dynamic storage behaviour itself in the absence of withdrawals from the dam. This also provides information on water storage extremes. A knowledge of such extremes are invaluable in realistic water management and, therefore, contributory to our ultimate objectives.

In the first instance, the sensitivity of storage is analysed with respect to variability in parameter estimates defined by the variance-covariance matrices in the earlier estimation. This is based on a stochastic-Monte Carlo simulation model for storage behaviour which is then further examined for variability with varying rainfall sequences. The former takes into account the uncertainty associated with the model by sampling from the distribution of parameter estimates generated by the refined IV-AML algorithm. The steps involved in such sampling have been dealt with in detail in Chapter 3.

4.4.1 Simulated Water Storage Distribution

Monte Carlo simulation in the present problem involves three parameters in all, viz. a_1, b_o and c_1. Other parameters in the rainfall filter (4.5) have been assumed not to vary. Given more rainfall and storage data, it would not be difficult to tune these to fairly precise values. Whereas the noise model parameter c_1 is sampled from a normal distribution, the former two are drawn from a bivariate normal distribution. This has been illustrated in Chapter 3; and the form of the sampling is shown by equation (3.27).

Now, by replacing in equation (3.27) the estimated means, standard deviations (variances) and covariance of a_1 and b_o parameters from Table 4.2, we have the sample values defined as:

$$\alpha_1 = 0.938 + 0.0028\ z_1$$

$$\beta_o = 0.687 + \frac{4.98 \times 10^{-5}}{0.0028}\ z_1 + \sqrt{(0.022)^2 + \frac{(4.98 \times 10^{-5})^2}{(0.0028)^2}} \cdot z_2$$

In a similar manner, values (γ_1) for parameter c_1 can be drawn such that:

$$\gamma_1 = 0.587 + 0.0853\, z_3$$

As in the original equation (3.27), z_1, z_2 and z_3 are independent standard normal variates.

By randomly selecting values α_1, β_o and γ_1 for the parameters a_1, b_o and c_1 each simulation run generates a forecast of water storage according to:

$$y_k = \frac{\beta_o}{1+\alpha_1 z^{-1}} \cdot u_k^* + \frac{1}{1+\gamma_1 z^{-1}} \cdot e_k, \qquad k = (1,\ldots,45)$$

After a sufficient number of simulation runs[1] are completed with the given year's rainfall data, the frequency distribution of y_k in each week, k is used to compute the mean and standard deviation of storage for that rainfall year. These statistics for the whole set of weeks are used to generate a mean storage distribution with a confidence band. Although probability densities of y_k for each week can be built up, consideration is restricted here to two points in time: the third week of February and the third week of August. Water storage levels in these two weeks are of particular interest. The former marks the beginning of the dry season since this is the week after the harvesting and threshing of the wet season's rice crop and hence the first available week for beginning the dry season's cultivation. On the other hand, the third week of August is taken to mark the end of the dry season and has the lowest water storage in the dam for *in situ* uses.

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1 The computer program developed for simulation is given in Appendix C.

Figure 4.6 displays the simulated storage distribution and the probability densities of storage at the beginning (Week 19) and at the end (Week 45) of the dry season. The upper and lower bounds as shown by the dashed lines are, respectively, one standard deviation above and below the average storage distribution function. Thus, if the simulated outputs could be assumed to be samples from a normal distribution, then these bounds would represent the 67 per cent confidence limits. Clearly, standard deviation measures provide an index of the variability or sensitivity of the forecast. But here the standard deviations are uniformly quite low and week to week variations are not significant in absolute terms. This is also evident from Table 4.3. The coefficient of variation (C.V)[1] of storage seems to provide greater insight into the variability. Table 4.3 shows that, in general, the C.V values are relatively higher for weeks of lower storage which are generally in the dry season; whereas, the higher storage weeks of the wet season are associated with lower C.V's. Since a higher variability also means a lower reliability, it implies that forecast of storages for the dry season are less reliable than those for the weeks in the wet season. This is particularly interesting because the forecast mean distribution from simulation compares well with the storage levels actually observed as shown in Figure 4.7. That is, the C.V values provide useful supplementary information to these latter forecasts which should not be viewed in isolation.

These simulation results have made use of the actual rainfall which was used earlier to identify the time series model.

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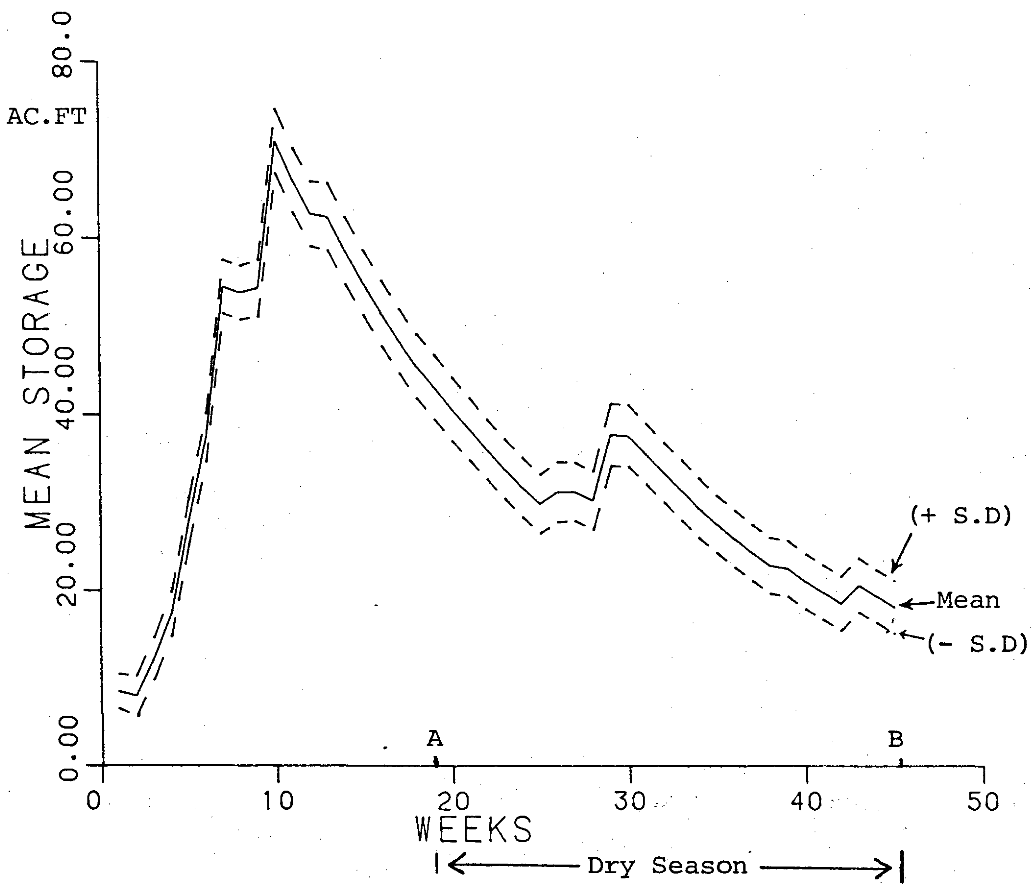
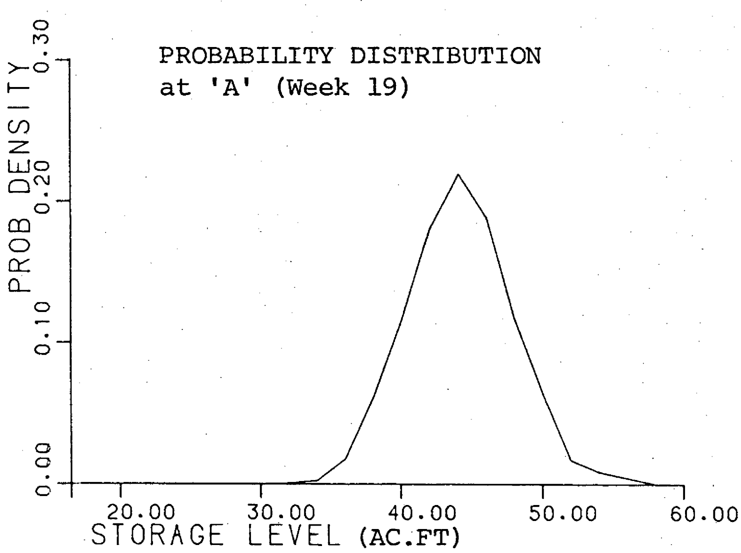
1 C.V is defined as the ratio of the standard deviation (S.D) to the mean.

FIGURE 4.6

SIMULATED WATER STORAGE DISTRIBUTION
AND PROBABILITY DENSITIES FOR 1976/77





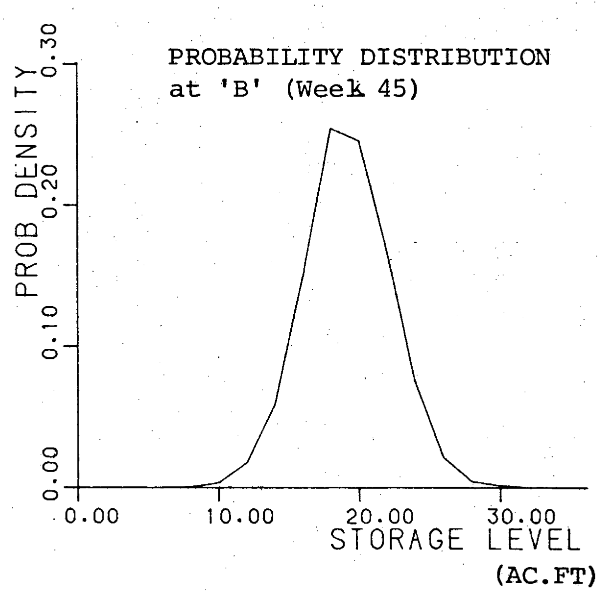


TABLE 4.3

ACTUAL AND SIMULATED WEEKLY WATER STORAGE, AND RELATED STATISTICS, 1976/77

Week	Water Storage (Ac.ft)				Week	Water Storage (Ac.ft)			
		Simulated					Simulated		
	Actual	Mean	S.D	C.V		Actual	Mean	S.D	C.V
1	8.5	8.49	1.97	0.232	24	30.5	31.75	3.38	0.106
2	7.5	7.97	2.33	0.292	25	28.5	29.87	2.33	0.078
3	10.0	12.23	2.41	0.197	26	30.5	31.23	3.41	0.109
4	12.8	17.30	2.52	0.146	27	29.3	31.24	3.32	0.106
5	30.0	27.79	2.66	0.096	28	26.8	30.27	3.32	0.110
6	32.0	37.39	2.74	0.073	29	32.8	37.75	3.51	0.093
7	48.5	54.47	3.01	0.055	30	32.8	37.61	3.49	0.093
8	48.0	53.82	3.05	0.057	31	31.3	35.56	3.43	0.096
9	46.0	54.32	3.20	0.059	32	30.8	33.46	3.38	0.101
10	64.0	70.99	3.62	0.051	33	29.3	31.47	3.44	0.109
11	64.3	66.58	3.64	0.055	34	29.0	29.45	3.38	0.115
12	61.5	62.76	3.66	0.058	35	27.2	27.62	3.26	0.118
13	60.0	62.43	3.81	0.061	36	25.2	25.91	3.26	0.126
14	56.0	58.57	3.80	0.065	37	23.2	24.32	3.19	0.131
15	52.0	54.97	3.69	0.067	38	21.7	22.87	3.19	0.139
16	48.5	51.49	3.72	0.072	39	23.2	22.56	3.20	0.142
17	45.7	48.32	3.65	0.076	40	21.7	21.08	3.16	0.150
18	43.5	45.50	3.65	0.080	41	20.2	19.85	3.13	0.158
19	41.0	43.14	3.67	0.085	42	18.7	18.59	3.08	0.166
20	39.0	40.63	3.61	0.089	43	22.0	20.64	3.10	0.150
21	37.5	38.37	3.52	0.092	44	20.4	19.38	3.02	0.156
22	35.0	36.03	3.40	0.094	45	19.2	18.16	3.01	0.166
23	32.3	33.83	3.38	0.100	-	-	-	-	-

Notes: a) This covers the period of 45 weeks from 14 October 1976 to 24 August 1977; and weeks 19 and 45 mark the beginning (A) and the end (B) of the dry season.

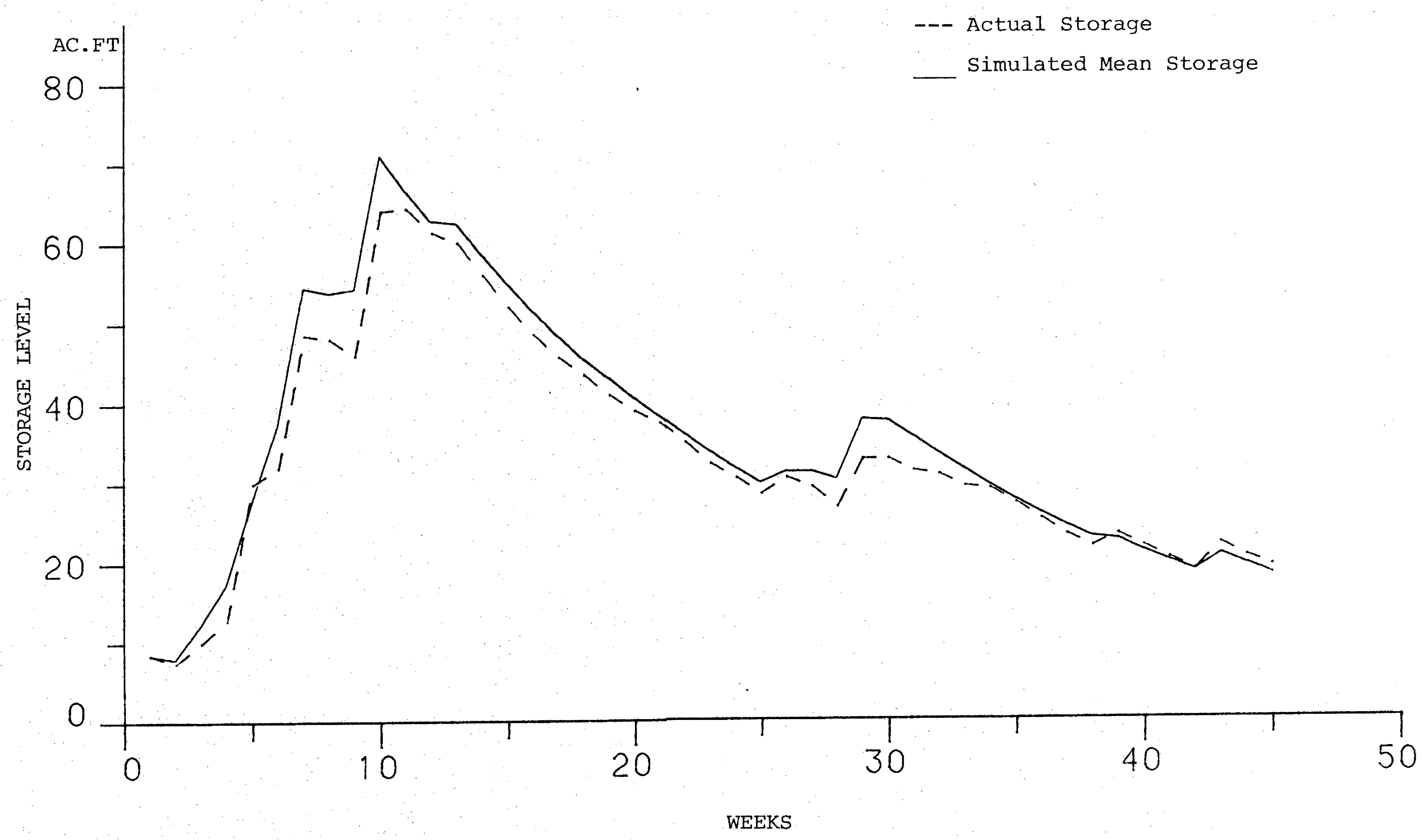
b) Simulation results are based on 2055 repetitions.

c) S.D and C.V denote standard deviation and coefficient of variation respectively.

FIGURE 4.7

A COMPARISON OF ACTUAL STORAGE FOR 1976/77 AND THE FORECAST

MEAN OF SIMULATION OUTPUT AFTER 2055 REPETITIONS



Also, the rainfall year, 1976/77 used to identify the model (4.7) and generate the simulation results so far, is regarded as representative of an average type of year. We will be considering wetter and drier rainfall years subsequently.

It is of general interest to examine the storage properties at the beginning and at the end of the dry season. They can also be useful in pre-season planning of the possible extents of rice cultivation. Statistical properties of storage at these two points in time are presented in the middle column of Table 4.4. An examination of this table reveals that the storage at the beginning of the dry season ranges from 32 to 58 acre feet with a mean of 43 acre feet which is very close to the mode. On the other hand, the storage at the end of the season appears to have a higher variability relative to the magnitude of the mean storage. Although the mean is only 18 acre feet, it has a range of 22, from 8 to 32 acre feet. This is reflected in the C.V which is 17 per cent as against a 9 per cent for the storage at the beginning of the season. The variability is greater when considerations are given to the variations in rainfall.

4.4.2 Probabilistic Forecasts of Storage and Rainfall Extremes

As noted earlier, our analysis in the previous sections has been with respect to an 'average' rainfall year; and it chiefly reflected the uncertainty associated with the time series model. An understanding as to the possible range of storage behaviour with respect to rainfall variability can be gained by entering extreme rainfall years in simulation. Two extreme years are considered

TABLE 4.4

SOME STATISTICAL PROPERTIES OF THE SIMULATED WATER STORAGE AT THE BEGINNING (A) AND AT THE END (B) OF THE DRY SEASON FOR THREE DIFFERENT RAINFALL YEARS

Statistics	Dry Year 1972/73	Average Year 1976/77	Wet Year 1959/60
Annual Rainfall in m.m	1074	1430	1883
Beginning of the Dry Season ('A')			
Water Storage (Ac.ft)			
Mode	37	44	63
Range	24-36	32-58	54-82
Mean	27.4	43.1	65.4
C.V	0.111	0.085	0.068
End of the Season ('B')			
Water Storage (Ac.ft)			
Mode	10	20	40
Range	0-20	8-30	26-58
Mean	9.6	18.2	40
C.V	0.281	0.166	0.108
Ratio of Mean Storage at 'B' to that at 'A'	0.35	0.42	0.61

for this purpose: namely, 1959/60 and 1972/73, representative of a wet year and a dry year respectively.

Simulated storage distributions for these years' rainfall series are graphed in Figures 4.8 and 4.9 in a manner similar to Figure 4.6. They exhibit remarkable overall differences from Figure 4.6. The differences in the probability distributions of storage at the two weeks of interest are compared in Table 4.4. The mean storage at the beginning of the season is 21 acre feet for the dry year. But it is as high as 65 acre feet for the wet year. As expected, the mean storage of 43 acre feet in the average year takes a position in between them. This is also true with respect to the other statistics such as the range and the C.V.

The simulation results of the wet year, dry year and the average year when considered together indicate the possible extremes in storage. Thus the storage at the beginning of the season seems to vary from around 24 to 82 acre feet, whilst the range is 0 to 58 acre feet for the third week of August which is taken to mark the end of the dry season.

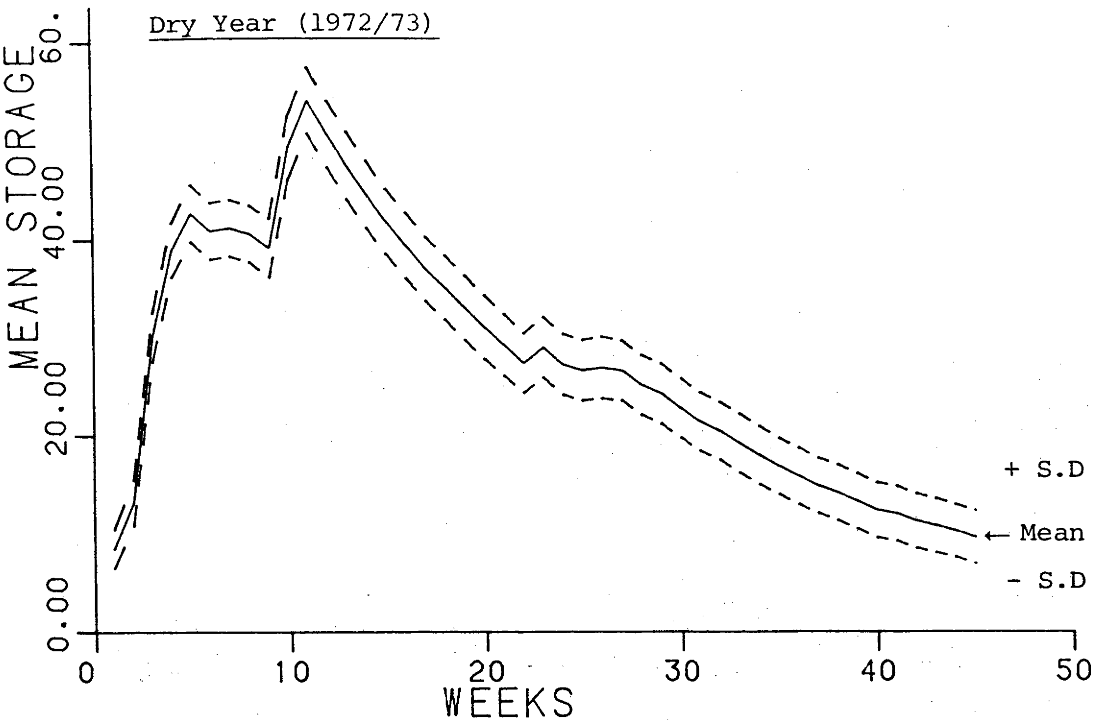
4.4.3 An Analysis of Forecast Mean Storages at the beginning and at the end of the Dry Season

The main value of information about the probable storage levels at the beginning and at the end of the season would seem to be its use in pre-season planning exercises. Undoubtedly, planning for dry season cultivation can be aided by:

(i) a knowledge on the preferred minimum level of storage to be maintained in the dam; and

FIGURE 4.8

SIMULATED MEAN STORAGE DISTRIBUTION AND 67 PERCENT CONFIDENCE BANDS FOR A DRY AND A WET YEAR



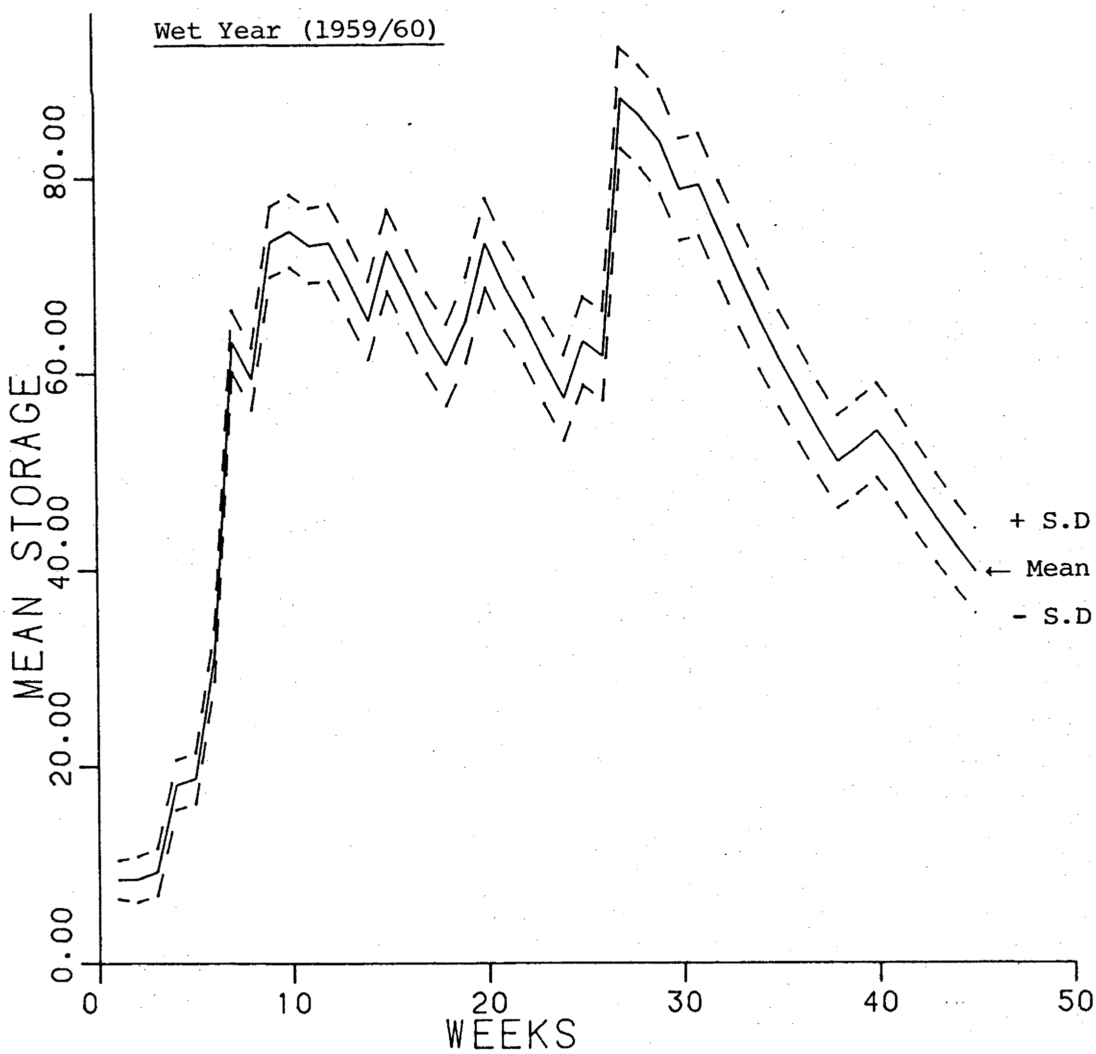
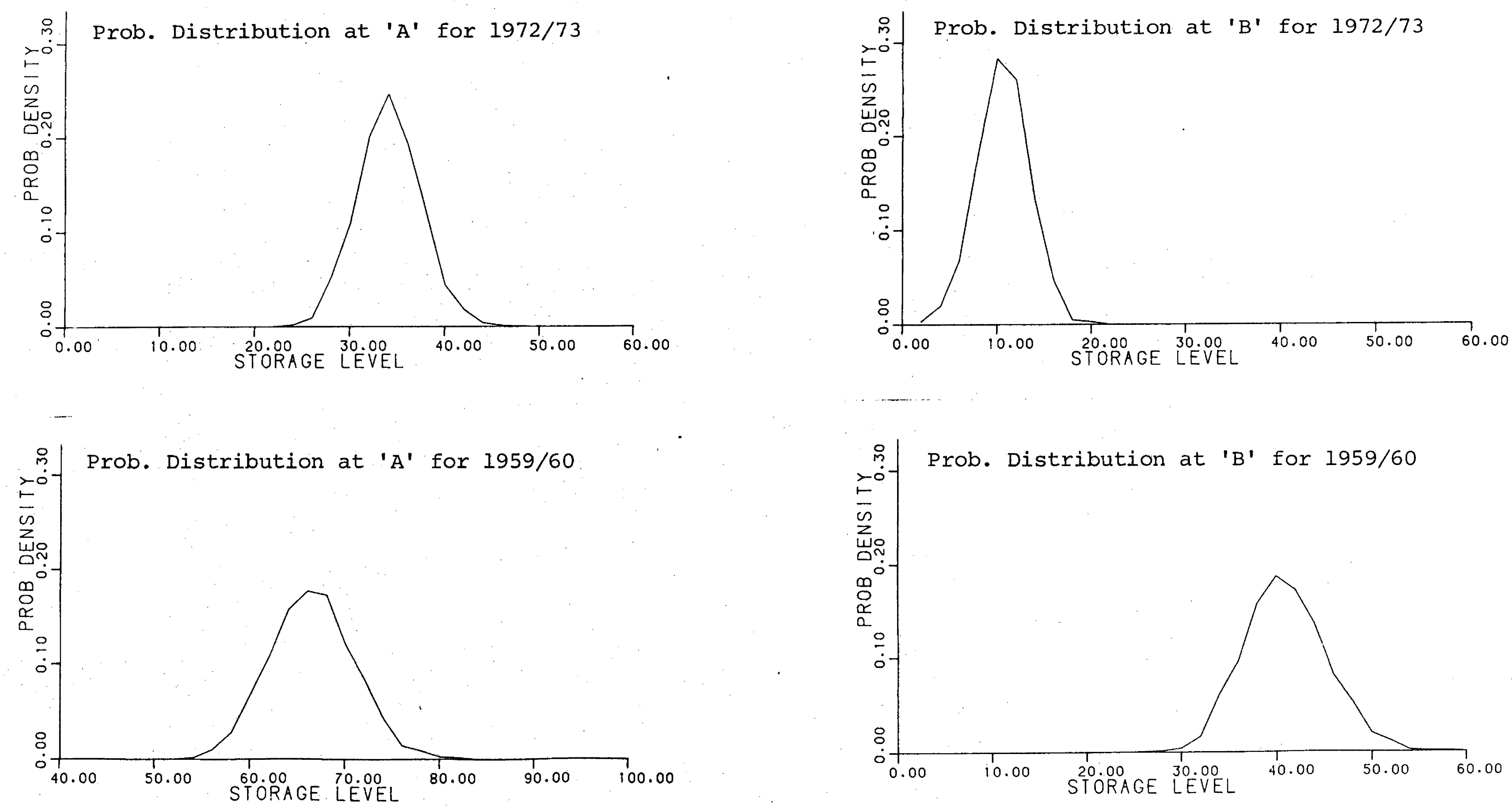


FIGURE 4.9

PROBABILITY DISTRIBUTION OF STORAGE AT THE BEGINNING (A) AND
AT THE END (B) OF THE DRY SEASON FOR DRY (1972/73) AND WET (1959/60) YEARS



(ii) forecasts of storage levels at the two points in time (the third week of February and the third week of August).

With regard to the former, the small dam community under review seems to prefer the storage to be at least around 10 acre feet, below which its *in situ* uses are apparently seriously impaired.

On the other hand, the use of forecasts for planning needs further consideration. The forecast of storage levels used above can be misleading unless consideration is given to a related aspect: namely, the uncertainty which is reflected in the coefficient of variation (C.V). Usually, the higher the C.V the higher the uncertainty that must be attached to the expected storage. The C.V and hence the uncertainty in storage levels seems to vary not only within a year but also between years. An examination of Table 4.4, for example, reveals some interesting trends among the wet, average and dry years. Uncertainty associated with the storage at the beginning of the season appears to decline as the year becomes wet and vice versa. This is reflected in the declining C.V from 'dry' to 'wet' through 'average' in Table 4.4. A similar trend is also evident with respect to storage at the end of the season. In other words, a wet year is generally associated with a relatively certain and high level of storage whereas the storage in the dry year is not only low but also highly uncertain. This indicates the need for caution in crop and irrigation planning *especially* in a dry year.

However, one advantage for the planner is that the forecast for the beginning of the season can be matched with the actual storage *before implementing the plan*. This is not possible with respect to storage at the end of the season. It is in this respect that any simple relationship between the storage forecasts between the two points in time will be valuable in planning terms. Such a relation, if it exists, can provide a 'rule of thumb' for planners.

The last row of Table 4.4 expresses the mean storage level at the end of the season as a ratio to that at the beginning. The ratio is 0.35 for the dry year whereas it is 0.61 for the wet year. In other words, the wet year appears to leave behind a proportionately higher amount of water at the end of the season in comparison to the dry year. In contrast, the dry year seems to share the worst of everything: lower initial storage, high uncertainty and a proportionately lower storage at the end. Such a tendency, if established, will have important implications for dry season's production, in particular the extent or acreage of rice to cultivate. A closer scrutiny is, therefore, felt worthwhile using forecasts for twenty different series (or years) of rainfall.

The probabilistic forecasts of storage at the beginning and at the end of the dry season for a set of 20 years are presented in Table 4.5 along with a breakdown of rainfall. To aid examination for a likely relationship, the dispersion of the mean storages at the two points in time is shown in Figure 4.10. In fact, it reveals that the wet and dry years considered earlier on are exceptions to the general trend, which seems to suggest a possible

TABLE 4.5

RAINFALL AND SIMULATED WATER STORAGE AT THE BEGINNING AND AT THE END OF THE DRY SEASON

Year	Rainfall mm		'Simulated' Water Storage in the Dry Season Ac.ft			
			Beginning (A)		End (B)	
	Wet Season	Dry Season	Mean	C.V	Mean	C.V
1959/60	855	1028	68.8	0.070	39.7	0.107
1960/61	1020	553	72.5	0.072	22.7	0.156
1961/62	928	409	51.4	0.081	14.0	0.209
1962/63	1103	625	61.0	0.072	23.9	0.145
1963/64	1403	432	67.0	0.078	16.0	0.194
1964/65	660	726	35.1	0.099	37.5	0.099
1965/66	1221	708	79.5	0.071	23.1	0.157
1966/67	1105	454	53.1	0.084	14.7	0.208
1967/68	939	480	43.7	0.094	13.6	0.214
1968/69	748	547	35.2	0.103	15.8	0.183
1969/70	884	493	65.6	0.070	19.1	0.173
1970/71	371	721	19.1	0.142	24.5	0.125
1971/72	989	364	51.3	0.085	13.0	0.230
1972/73	753	331	29.2	0.116	9.5	0.279
1973/74	780	488	48.1	0.080	16.9	0.176
1974/75	531	742	37.4	0.089	32.3	0.107
1975/76	533	374	28.8	0.116	18.8	0.161
1976/77	951	529	52.3	0.083	20.3	0.151
1977/78	1046	502	41.2	0.098	14.3	0.200
1978/79	849	217	53.1	0.088	12.4	0.242

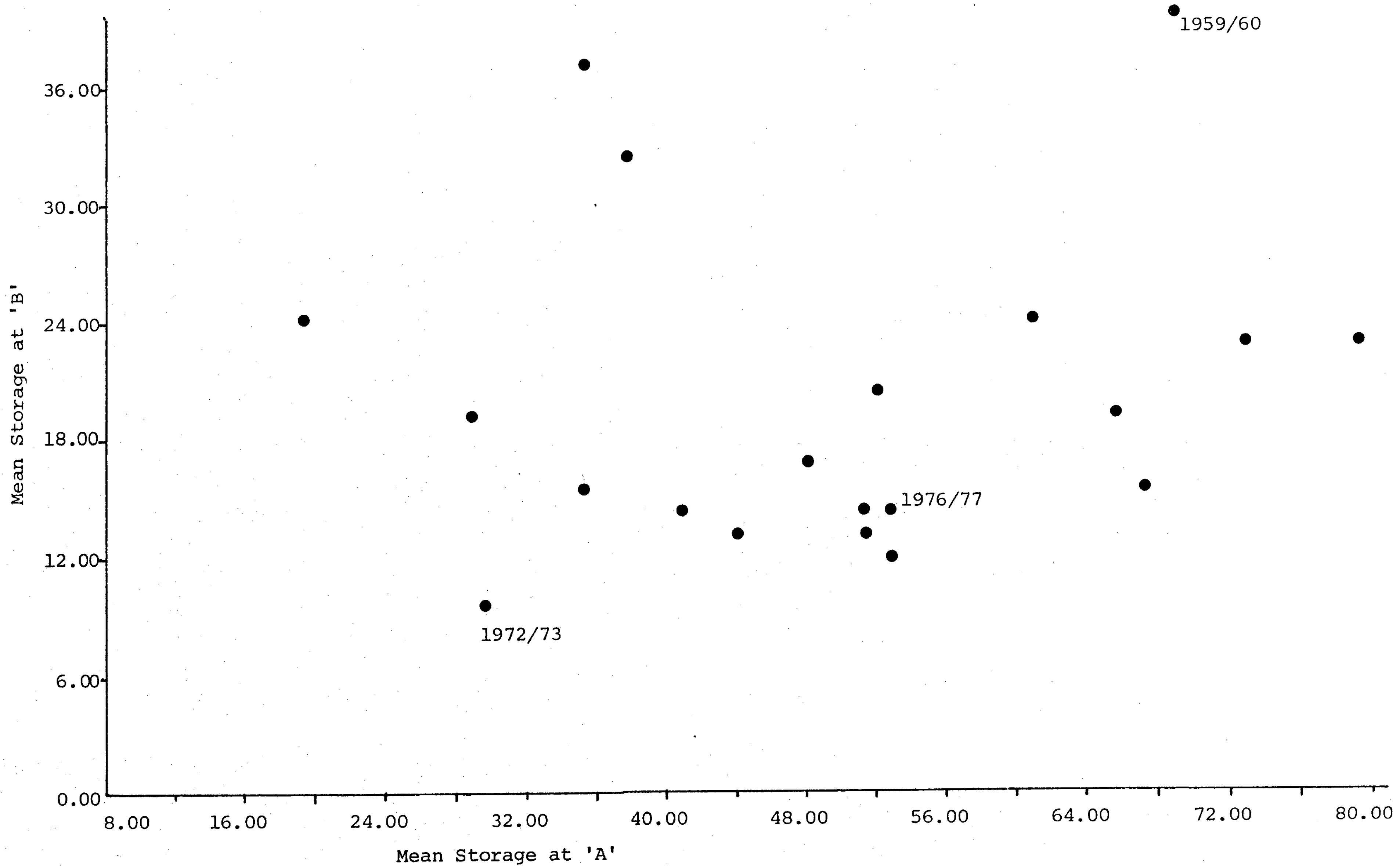
Notes:
a) The wet season's rainfall represents the aggregate for the first 20 weeks from the first week of September.

b) The third week of February and the third week of August, in respective order, mark the beginning and the end of the dry season.

c) Simulation results are based on 186 repetitions.

FIGURE 4.10

DISPERSION OF FORECAST MEAN STORAGE AT THE END (B)
AGAINST THAT AT THE BEGINNING (A) OF THE DRY SEASON



quadratic relationship[1] between the mean storage at the end of the season and that at the beginning. The minimum of season-end storage level is around 12 acre feet and corresponds to approximately 50 acre feet at the beginning. Higher initial storage than 50 acre feet, as expected, leads to higher levels of storage being left at the end. However, *in contrast to earlier conclusions*, lower initial storages also tend to leave relatively higher levels of storage at the end.

At least a part of the explanation for the above relationship seems to rest on the rainfall distribution. Obviously, the wet season rainfall determines the storage level at the beginning of the dry season. This, together with the rainfall during the dry season, is largely responsible for the storage at the end. Thus it appears that even a lower initial storage as a result of poor wet season rainfall leads to relatively higher year-end storage by an associated good dry season rainfall. Such a tendency, though not significant, is suggested by a regression analysis of the dry season rainfall against that of the wet season.[2] For the 20 years rainfall data, dry season and wet season rainfalls exhibit a negative, but not significant, relationship.

There is yet another aspect of the storage forecasts to be considered in relation to the distribution of storage. Figure 4.11 presents the frequency distributions of forecast mean storages, derived from Table 4.5, at the two points in time under consideration.

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1 However, it does not provide a satisfactory explanation statistically, since R^2 is only 0.37 for

$$W_e = 3.75 - \underset{(0.0307)}{0.0410}\, W_b + \underset{(0.0003)}{0.00045}\, W_b^2$$

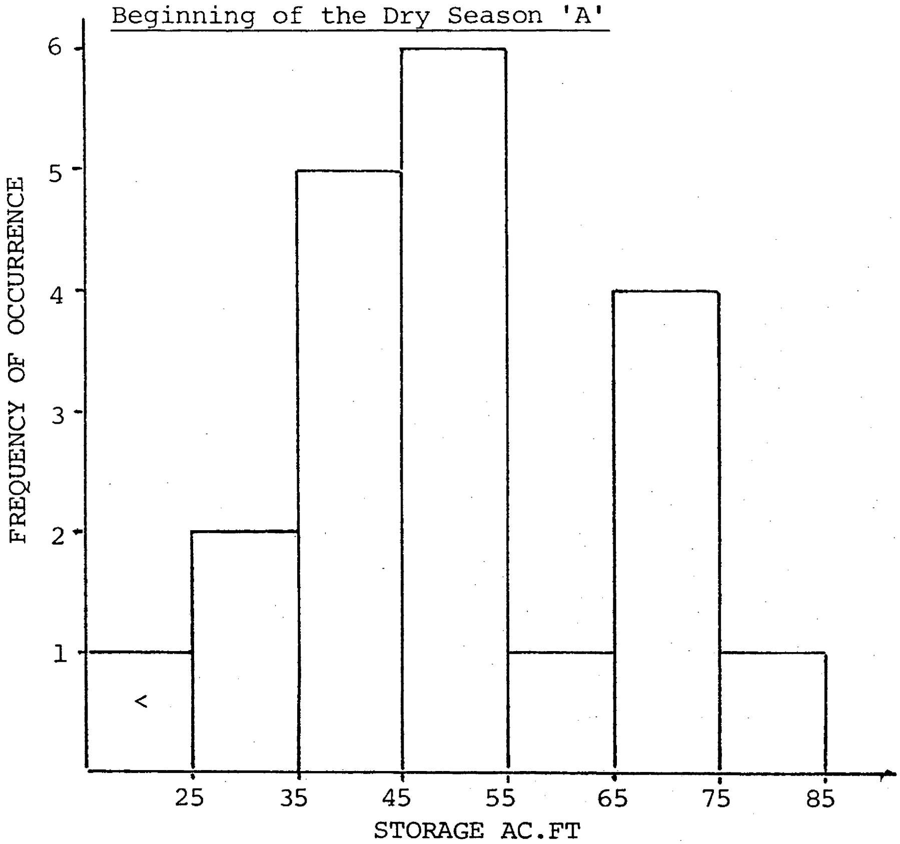
where W_b and W_e denote the forecast mean storage levels at the beginning and at the end of the dry season, respectively.

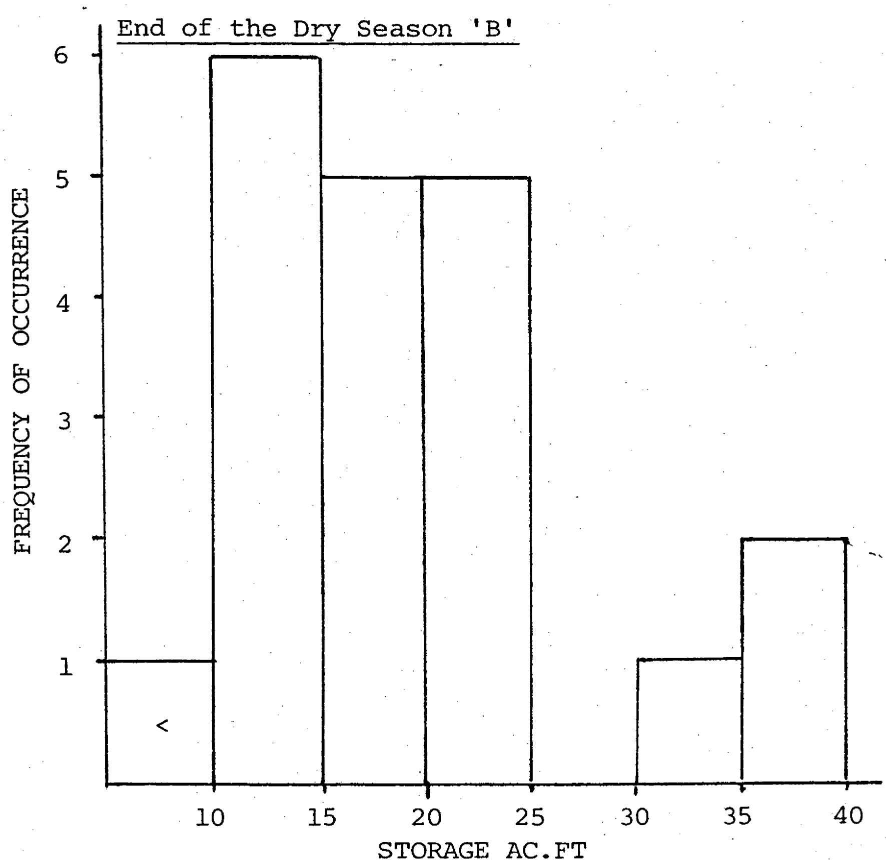
2 $R_d = 651.8 - \underset{(0.1701)}{0.1309}\, R_w$; correlation coefficient = -0.158

where R_d represents the total dry season rainfall;
R_w denote the rainfall during the wet season of 20 weeks from September to January;
and the figure in parenthesis is the standard error.

FIGURE 4.11

HISTOGRAMS OF FORECAST MEAN WATER STORAGE





As regards the storage at the beginning of the season, in seventy per cent of the years, it is less than 55 acre feet: of which, for 55 per cent of these years, the range is from 35 to 54 acre feet. The modal storage seems to lie in the range of 45 to 54 acre feet. As a whole, for these twenty years, the mean forecast storage at the beginning of the dry season works out to around 50 acre feet.

On the other hand, the frequency distribution of storage at the end of the dry season is notably different. For 80 per cent of the years, the mean forecast lies in the range from 15 to 25 acre feet with almost an equal probability of occurrence. The overall mean works out to approximately 20 acre feet. Although small in range, it must be kept in mind that this is in respect of expected values (or means). These discussions suggest that an accurate prediction of the dry season rainfall beforehand is very valuable for planning purposes.

To make a forecast of the storage at the end of the season just before the beginning of the dry season, there are two possible alternatives, viz.,

(i) to make use of a scatter-gram similar to Figure 4.10, if the year seems representative. Whether a year is representative or not can be found by matching the actual storage at the beginning of the season with the forecast made using the rainfall observation for the preceding wet season. If the two do not match well, then the year may be considered non-representative, in which case

(ii) the forecast can be made by referring to the dry season rainfall of a year that exhibits similarity to the current year in terms of wet season's rainfall. This is a form of 'pattern recognition'. The difficulty in this approach lies in selecting the similar year from past rainfall records.

However, such attempts can provide only a rough estimate of the storage at the end of the season. Also, although such forecasts may be useful, they do not address the question of how water has to be withdrawn. It is, in fact, the temporal nature of 'how' the water has to be withdrawn, *especially* with the dynamic supply, that constitutes the theme of the present research. This is discussed next using the stochastic simulation model developed in this chapter.

4.5 Summary of Results

The results presented in this chapter could be summarised as follows:

(i) For time series modelling of water storage in the dam, weekly samples of rainfall and water levels proved to be adequate. Altogether 45 samples were used for 1976/77, together with weekly mean ambient temperatures.

(ii) A linear effective rainfall-storage model was obtained by pre-filtering the actual rainfall input to compensate for current temperature and soil moisture levels. The soil moisture compensation required exponential weighting into the 6 past weeks and also a slight differential weighting for the dry and wet season rainfalls. The compensation for temperature effects were simple and incorporated a function based on the difference of the weekly mean temperatures from a notional maximum of 100°F.

(iii) After modification of the input, the identified TF model for storage explained 97 per cent of the variation in the data for 1976/77, with just two parameters. Physically, the model interprets the current week's storage level as equal to 68 per cent of that week's effective rainfall plus 94 per cent of the preceding week's water level in the dam. The residual sequence unexplained by the process was found to be purely autoregressive of the form ARMA(1,0). In the estimation of parameter values for the system and the noise models, both the refined and basic IV-AML algorithms provided good parameter definition with the refined IV-AML parameter variances lower as expected.

(iv) Monte Carlo simulation was performed employing the estimates and variance-covariances obtained from the refined IV-AML. Probabilistic forecasts were then made for the storage for 1976/77, the year for which the time series model itself was identified. In particular, 67 per cent confidence bands for mean storage and also probability density functions for all possible storage levels at the beginning and at the end of the dry season were provided. Given that the year 1976/77 was representative of an average rainfall year, a wet and a dry year's rainfall data were also entered in simulation separately to highlight the extremes in storage. To investigate possible relationships between the storage at the beginning and at the end of the dry season, mean storage forecasts for 20 rainfall years were also examined. A method of approximate prediction of the dry season rainfall emerged as an important requirement for more successful pre-season planning of cultivation in the rice land.

CHAPTER 5

WITHDRAWAL POLICY, STRATEGIES AND EVALUATION

This chapter is devoted to an empirical demonstration of how the choice of an optimal withdrawal strategy can be determined for an early maturing rice crop. In Section 5.1, irrigation policy is defined to provide a background for the formulation, in Section 5.2, of decision rules for our specific policy, namely the minimal supplementary irrigation policy. Section 5.3 defines various irrigation (or withdrawal) strategies and deals with the comparison of their performance. In Section 5.4, the same set of strategies is again examined, entering different rainfall years as input to our simulation model, and the consistency of the results is indicated. In the light of these findings, the prevalent practice (or time) of commencing dry season rice cultivation is assessed in Section 5.5. The overall results are summarised in Section 5.6.

5.1 Withdrawal Policy

Withdrawal is defined, and treated in this dissertation, as the process by means of which the dam water is released over time for application to or irrigation of crop(s). Alternatively, it is a process whereby each stage of the crop is allowed the use of the dynamic dam water supply. A policy designates guidelines or courses of action to fulfil certain specified objectives. A withdrawal or irrigation policy for rice cultivation near a small dam site,therefore, defines an action system or a set of actions designed to guide or govern the release of water over the cropping season with a view to attaining the adopted objective, namely ensuring a rice crop.

In addition to specifying the adopted objective initially, a complete description of such a policy should also establish the action system to effect the temporal irrigation. Finally, the institutional framework to facilitate the operation of the policy should be addressed. Fortunately, this third aspect does not pose a serious problem here, since the dam and rice land are small and favourable community institutions exist.

The approach to our action system has been outlined in Chapter 2 as a minimal supplementary irrigation policy. This can only be defined by a set of decision rules, since at each stage of the crop the withdrawal has to be decided in relation to the incident rainfall which is variable.

5.2 Decision Rules for Irrigation of Rice

It has been argued in Chapter 2 that the water demand for early maturing rice varieties can be regarded as intraseasonally constant. Also, having decided that the level of application of water should be such that a crop of rice is ensured, then the irrigation levels can be defined only by a set of decision rules. This is because although the demand for water is constant throughout, the incident rainfall is variable. In other words, the action system or the set of decision rules represents the pragmatic form of the minimal supplementary irrigation outlined earlier. But, in addition, the decision rules must incorporate other agronomic considerations as well. Thus for crop establishment, for example, allowance should be made for wet land preparation.

Irrigation may be required for a period of eleven weeks commencing on the third week after the initial withdrawal for land preparation. Thus two weeks is allowed for land preparation, as it is practised. It is believed that 5 acre feet of water is adequate for the preparation of the whole 30 acres of rice land below our dam. But the land preparation could be done without any water from the dam, provided the rainfall has been in excess of 80 mm during the whole week. Further, if the rainfall has been less than 80 mm in that week, then that amount is also taken into account. The latter is given a weight of 0.2 as against 0.8 for the preceding week's rainfall in the computation of the aggregate contribution of rainfall to land preparation. Every 10 mm of this aggregate rainfall is treated as capable of saving 1 acre feet, out of the total 5 acre feet withdrawal from the dam.

During the course of crop growth, which encompasses 11 weeks of water application, a minimum of 13 acre feet of water is felt to be the likely requirement. A 13 acre feet application of water for 30 acres of rice land might sound very low. But this is based on the author's field experience in the small dam situation under consideration. One important hydrological reason for such a low water requirement has been advanced earlier in Chapter 2: i.e., an incessant seepage influence from the dam. In addition, it is felt desirable to consider every week for supplementary irrigation. This then conforms with popular notions regarding crop sensitivity to shortage of water. In this regard, it is worth noting the following comment of Chandler (1979, p.42):

> 'Under tropical conditions, the growth of the rice crop usually suffers from inadequate water unless rain or irrigation occurs every week or ten days'.

Irrigation decisions at each week are based on the rainfall during the two preceding weeks. An irrigation, if necessary, considers a withdrawal of either 1 or 1.5 acre feet. An irrigation is considered only if the rainfall in the preceding weeks is not in excess of 10 mm. A 10 mm rainfall is approximately *equivalent* to 0.4 acre inch per acre of water application for the whole rice land. If the rainfall happens to be within the range of 5 to 10 mm, then a withdrawal of 1 acre feet is made. On the other hand, if it is below 5 mm, then a higher amount 1.5 acre feet is withdrawn for application. Consideration is also given to likely situations that arise with occasional rainstorms. If such a rainstorm is in excess of 80 mm, it can meet the requirements of rice for the following two weeks.

Using the approximate figures that (a) 1 inch = 25 mm and that (b) the total extent of the rice land is 30 acres, as in our case, the arithmetic[1] of irrigation is as follows:

10 mm of rainfall at the site is equivalent to $\frac{10}{25}$ or 0.4 acre inch/acre for the whole rice land.

An acre foot of dam water spread over the whole rice land equals $\frac{12}{30}$ or 0.4 inch, or 0.4 acre inch/acre

Similarly, 1.5 acre feet of water is equivalent to 0.6 acre inch/acre

Thus, 1 acre foot of withdrawal plus 5 mm rainfall

$$\equiv 0.6 \text{ acre inch/acre}$$

and 1.5 acre feet of withdrawal plus 0 mm rainfall

$$\equiv 0.6 \text{ acre inch/acre}$$

Thus, it may be noted that the irrigation policy sets aside some allowance for conveyance losses whilst ensuring, say, a minimum of

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1 These simple rules which consider only the preceding 2 weeks of rainfall could be replaced by a function like the soil moisture index in equation (4.2), which has an infinite memory. Of course, such an index would need to be validated and would involve an added complication in practice.

0.5 acre inch or so. It may, therefore, be surmised that the *minimum* level of application of water for rice in our situation is approximately 0.5 acre inch per week for the irrigation period of 11 weeks. In actual fact, it will be seen that the total amount of water applied is much higher due to the erratic nature of the rainfall.

However, it must again be reiterated that the decision rules outlined above are evolved from heuristic reasoning in an attempt to simulate the actual field situation. The process draws heavily from the personal observations of the author while such a minimal supplementary irrigation policy was implemented during the period 1976 to 1979 at the specific dam site. But withdrawal data available for the 1976/77 dry season are not strictly comparable with the policy outlined here for two reasons:

(a) there was a mid-season cultivation of beans and pulse crops on approximately 5 acres of the rice land; and

(b) in the dry season again a mixture of crops were tried using the dam water, but not with very much success. This was because the crops were affected by extreme wet conditions resulting from rainstorms, which, in a way, strengthened the choice of rice as the dry season crop.

In the succeeding two years, rice only was cultivated in the dry season. Unfortunately, actual storage and withdrawal data for these years are not available for comparison.

Nevertheless, in the formulation of the policy and decision rules, the emphasis clearly should be to explicitly account for the contributions of rainfall towards the water application (or supply) for the rice crop. Furthermore, any inaccuracy in setting the minimum level of application is not likely to affect the overall conclusions with respect to our specific objective: i.e., the time of planting of the dry season crop so as to minimise the withdrawal. This is because the conclusions are to be drawn from relative assessment of different ways (or times) of implementing the policy rather than the policy *per se*. In other words, referring back to the LP formulation of the problem in Section 2.5 in Chapter 2, it can be seen that altering the value of the resource level 'b' in (2.1) will not change the optimal solution.

The different ways (or times) of implementing the withdrawal policy are discussed and evaluated in the next section under the heading 'strategies'.

5.3 Strategies: Implementation and Evaluation

In all, eight strategies for implementing the water management policy are considered. Each strategy is defined by the week which marks the commencement of the dry season crop activity. Possibly, it is also the week in which the first withdrawal from the dam is made. For convenience, each strategy is named after the month and the week in which it commences: for example, M2 - for the strategy that commences on the second week of March. The strategies considered for analysis commence in the weeks between mid February to mid April; and they are F3, F4, M1, M2, M3, M4, A1 and A2. Strategies that commence later than A2 fail to make complete use of the dry season rainfall, and are therefore inefficient and easily eliminated from further consideration. On the other hand, strategies earlier than F3 are difficult to implement since the land and other resources would still be committed to the wet season's crop. However, in the analysis, F1 and F2 are also included to examine for possible advantages of the early commencment of crop activity.

The general approach to implementing a strategy is outlined in Section 3.3 of Chapter 3. Essentially, it involves the incorporation of both the decision rules for supplementary irrigation amounts and the sowing strategy or the time of commencing the cultivation in the computer program for simulation. Each strategy is implemented separately. In order to economise on computer time, a lower level of accuracy is opted by choosing to carry out only 186 simulations for each strategy.

Each strategy is evaluated by observing the probabilistic forecast of storage with the strategy being implemented. The statistics chosen to compare the different strategies include, in addition to the visual observation of forecast storage:

(a) the expected or mean storage level at the end of the dry season;

(b) the expected total withdrawal or irrigation over the season; and

(c) the probability of the dam becoming empty at the end of the season.

Essentially, in addition to minimising the total withdrawal from the dam, the probability that the dam will not empty before the end of the season must be maximised.

The results for the different strategies are presented in Table 5.1. Withdrawals associated with the different strategies

TABLE 5.1

PERFORMANCE OF THE DIFFERENT STRATEGIES OF WITHDRAWAL FOR 1976/77

Strategy	Amount of Withdrawal (Irrigation) Acre Feet	Simulated Storage at the End of the Dry Season		Probability of Emptying the Dam
		Mean	S.D	
F1	13.50	4.47	3.03	0.081
F2	13.47	4.54	3.01	0.075
F3	13.08	4.89	2.94	0.043
F4	12.71	5.28	2.92	0.038
M1	13.48	4.51	3.04	0.081
M2	13.28	4.77	2.99	0.059
M3	13.58	4.37	3.01	0.091
M4	13.50	4.47	3.03	0.081
A1	14.60	3.37	2.95	0.156
A2	13.10	4.86	2.93	0.043

Notes: a) The strategies are defined by the *month and the week* in which the land preparation is commenced, or when the first withdrawal of water from the dam is considered for the dry season's rice cultivation.

b) Results are based on 186 simulations for each strategy.

range from 12.71 to 14.60 acre feet, the lowest quantity being associated with F4. With reference to the amount of withdrawal, the strategies can be arranged as follows:

$$F4<F3<A2<M2<F2<M1<F1 = M4<M3<A1 \qquad (5.1)$$

It follows then that F4 and F3 are the optimal and second-best strategies in terms of withdrawal.

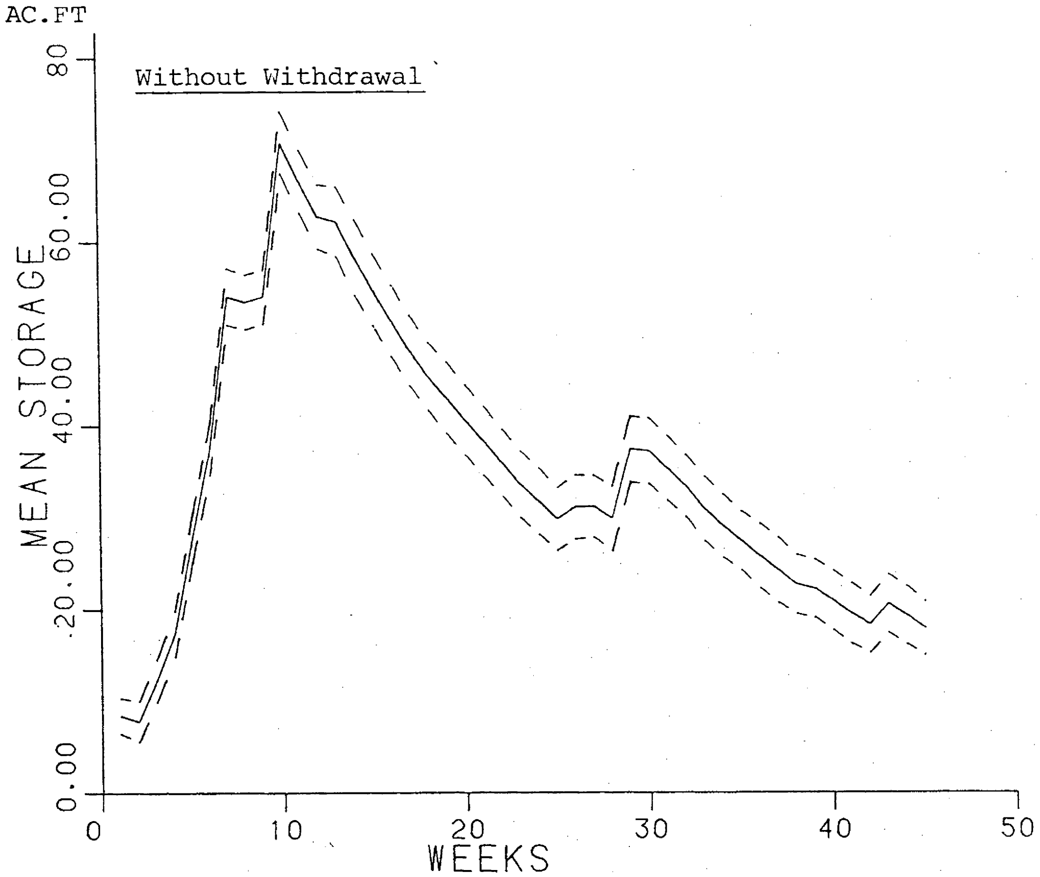
The storage criterion also confirms the above ranking, F4 being associated with the highest mean storage of 5.28 acre feet at the end of the dry season. Therefore, F4 is the optimal strategy that defines the commencement of the cropping season on the fourth week of February. The nature of the withdrawal for this strategy and its effect on the probabilistic forecast of storage is graphed in Figure 5.1. The depletion towards the later stages indicates the possibility that the dam will run completely dry, given the uncertainties associated with the model. In fact, the optimal strategy, F4, does have a probability, although low, of draining the dam empty (0.038).

The mean storage criterion and the probability of emptying the dam can be effectively combined by considering the cumulative probability distribution (CPD) of storage at the end of the season. The CPD for a few of the strategies are presented in Table 5.2. In Figure 5.2, the cumulative probability distribution for two of the strategies are graphed. The one to the right is associated with the highest mean storage and the lowest probability of emptying the dam. It corresponds to the optimal strategy, F4 which has 0.038 probability of emptying the dam. Similar, but opposite, reasoning suggests that the CPD furthest to the left represents the worst strategy, Al. The CPD's of the rest of the strategies in the respective order of their rank (5.1) lie between the CPD of F4 and that of Al. They are of course not shown in the figure for the sake of clarity.

Another observation is that the mean storage level at the end of the dry season for strategy F4 is only 5.28 acre feet, which

FIGURE 5.1

COMPARISON OF WATER STORAGE BEHAVIOUR WITH AND WITHOUT WITHDRAWAL (STRATEGY F4)



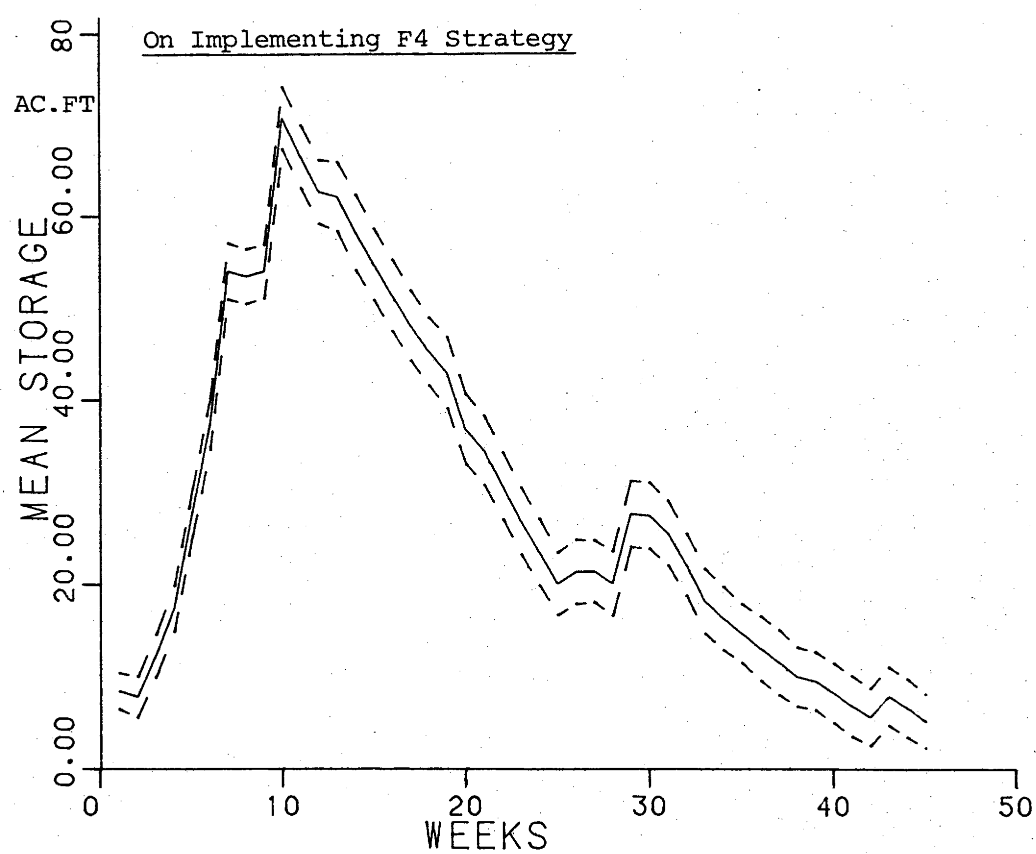


TABLE 5.2

CUMULATIVE PROBABILITIES OF STORAGE AT THE END OF THE DRY SEASON FOR VARIOUS STRATEGIES OF WITHDRAWAL

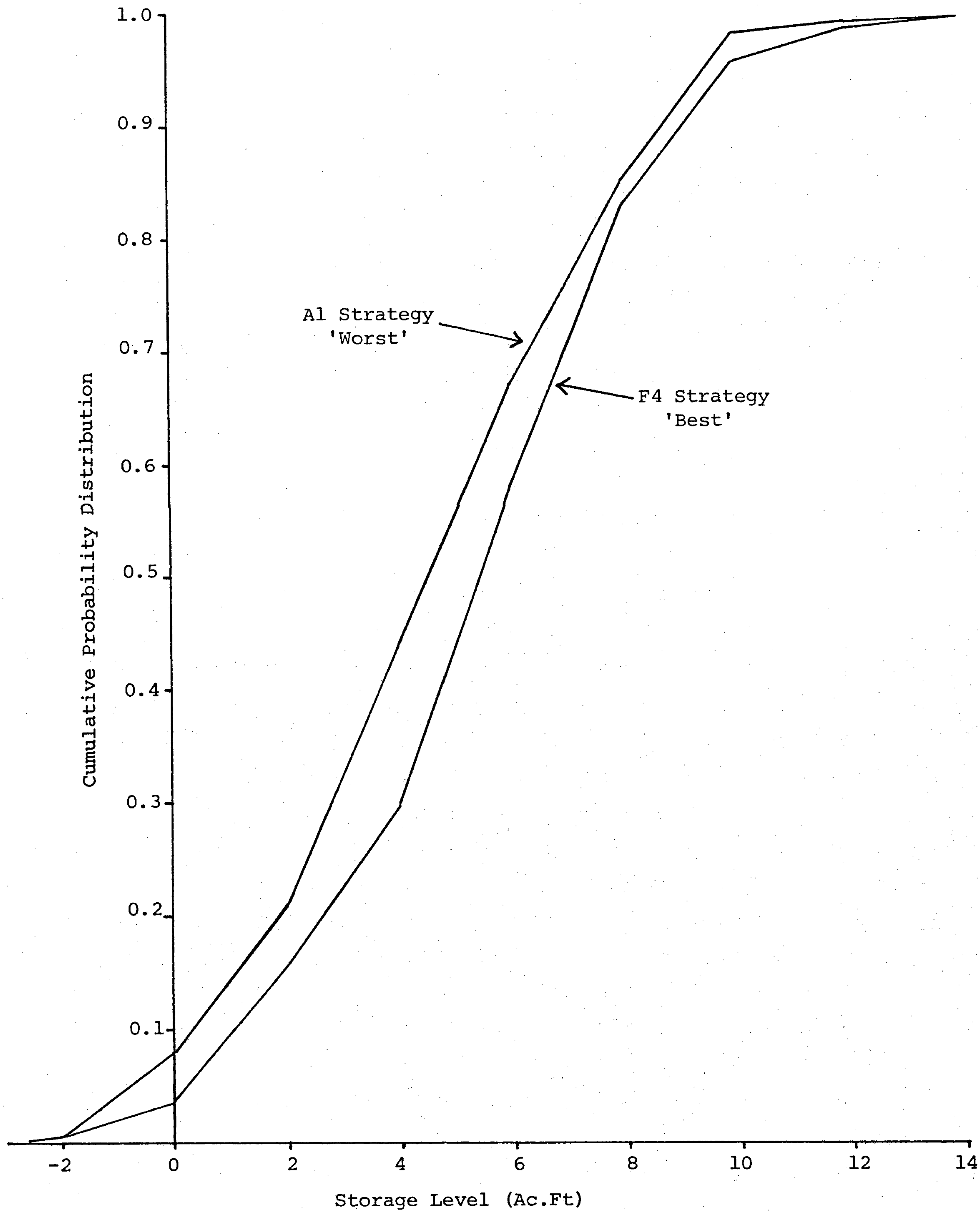
Storage Levels Acre Feet	'Best' (4th Week of Feb)	'Second Best' (3rd Week of Feb)	'Worst' (3rd Week of March)
≤ -2	0.005	0.005	0.005
≤ 0	0.037	0.043	0.091
≤ 2	0.155	0.177	0.210
≤ 4	0.295	0.376	0.468
≤ 6	0.580	0.639	0.688
≤ 8	0.827	0.844	0.882
≤ 10	0.962	0.973	0.989
≤ 12	0.993	0.994	0.994
≤ 14	1.000	1.000	1.000

is well below the community's preferred minimum level of approximately 10 acre feet. This indicates that the chosen policy can affect the *in situ* benefits of the community if the whole extent of 30 acres is cultivated. In other words, it implies that the two objectives are in the competitive range of the benefit transformation curve discussed in Section 2.5 of Chapter 2. It may be taken to suggest a reduction in acreage to be cultivated especially if the effect on *in situ* benefits is drastic. However, this is not pursued in this dissertation.

Another conclusion to be drawn from these results is that there is no special advantage in very early commencement of the dry season

FIGURE 5.2

CUMULATIVE PROBABILITY DISTRIBUTIONS OF SIMULATED WATER STORAGE AT THE END OF THE DRY SEASON WITH DIFFERENT WITHDRAWAL STRATEGIES



cropping. It also has implications for wet season cultivation in that it cannot be delayed. For instance, if for any reason the planting of the wet season rice is to be delayed, then the lost time needs to be balanced by the choice of an appropriate short age variety of rice so as to finish the harvesting by the third week of February. However, these results pertain to the year 1976/77 and the next section considers different rainfall years.

5.4 Optimal Strategy in a Variable Rainfall Environment

The variability in rainfall and its effect in storage forecasts has been seen in the previous chapter. Considering such variabilities, it is necessary to examine the strategies for their performance under different rainfall years before generalisations can be made regarding optimality. For this purpose twenty rainfall series (or 20 years) are considered separately to observe the performance of the selected ten strategies. The optimal strategy and related statistics for all the years are presented in Table 5.3.

The year to year variations in both withdrawal and end-of-season storage are remarkable. Although comparison across years can be misleading, it is interesting to note these variations. For instance, the withdrawals range from 5 to 16.5 acre feet, whilst the variation is even more striking in the storage criterion. Also, the dam is run completely dry for two years out of twenty. But, it may be noted that these two years are very 'dry' years in terms of rainfall, as would be expected.

However, in general the policy and the F4 strategy seem to have a very low probability of emptying the dam; for fourteen out of the twenty years the probability of emptying the dam is lower than 0.05.

TABLE 5.3

OPTIMAL IRRIGATION STRATEGY AND ITS PERFORMANCE

IN DIFFERENT RAINFALL YEARS

Year	Rainfall mm	Optimal Strategy	Withdrawal Ac.ft	Simulated Year-End Storage (Ac.ft)		Probability of Emptying the Dam
				Mean	S.D	
1959/60	1883	F4	7.50	32.15	4.25	0.000
1960/61	1573	F3	5.00	17.74	3.54	0.000
1961/62	1337	F3	10.50	3.51	2.92	0.151
1962/63	1728	F4	7.23	16.58	3.47	0.000
1963/64	1835	F4	10.97	5.09	3.11	0.043
1964/65	1386	F3	5.72	31.74	3.73	0.000
1965/66	1929	F3	10.33	12.72	3.62	0.000
1966/67	1559	F4	8.44	6.22	3.06	0.005
1967/68	1419	F4	10.48	3.17	2.91	0.167
1968/69	1295	F4	9.17	6.80	2.89	0.005
1969/70	1377	F4	7.47	11.78	3.31	0.000
1970/71	1092	M1	6.10	18.49	3.06	0.000
1971/72	1353	F3	11.00	2.02	2.99	0.242
1972/73	1084	F3	13.24	-3.74	2.65	0.936
1973/74	1268	M1	8.00	9.01	2.98	0.000
1974/75	1273	M2	6.00	26.34	3.47	0.000
1975/76	907	F4	15.50	3.33	3.02	0.161
1976/77	1480	F4	7.46	12.94	3.07	0.000
1977/78	1548	F4	7.02	7.27	2.86	0.000
1978/79	1066	F3	16.54	-4.14	3.00	0.914

Notes: a) The strategies are defined by the *month and the week* in which the land preparation is commenced, or when the first withdrawal of water from the dam is considered for the dry season's rice cultivation.

b) Results are based on 186 simulations for each strategy.

c) Rainfall data entered in simulations here were obtained from a nearby Agrometerological Station and not at the dam site , where it is slightly different for 1976/77. Thus it may be noted that the results for this year do not match with that presented in Table 5.1

As regards optimality, it is interesting to note that the F4 strategy turns out to be optimal in ten out of twenty years whilst F3 is optimal for seven years. This indicates that our results are *reasonably* consistent. Further insight can be gained by an examination of Table 5.4 which displays the distribution of optimal and 'second best' strategies for all the twenty years examined. This reveals that out of the ten years in which F4 is non optimal, it turns out to be second-best for eight years. Taken together, F4 is either optimal or second-best in eighteen out of twenty years. As the second-best strategy, F3 appears in seven of the years. Quite interestingly, the optimal strategy for these seven years is F4.

TABLE 5.4

THE DISTRIBUTION OF OPTIMAL AND SECOND-BEST STRATEGIES FOR TWENTY YEARS

		Optimal Strategy				
		F3	F4	M1	M2	Total
'Second best' Strategy	F3	-	7			7
	F4	5	-	2	1	8
	M1		3	-		3
	M2	2			-	2
	Total	7	10	2	1	

In contrast, of the eight years in which F4 is second-best, for five years F3 turns out to be the optimal strategy. This clearly shows that F4 and F3 are the most desirable strategies overall, with

F4 the single most useful strategy for optimal management of water resources in the small dam.

The consistency of optimality for variations in rainfall adds a further dimension to the generality of our findings. Insensitivity of the results to slight variation in irrigation policy has been noted already.[1] Thus, irrigation or withdrawal strategies[2] which are essentially framed to best supplement incident rainfall for crop growth are reasonably comparable across policies. Our findings can, therefore, be used to assess the optimality of the strategy actually adopted by the community and this is pursued in the next section.

5.5 Assessment of the Prevalent Practice

It is not possible to comprehensively compare our irrigation or withdrawal policy with that actually adopted by the community since 1976. The major difficulty, as noted earlier, is the lack of precise data on withdrawals. In fact, we have to rely on our single year's data.

Withdrawals from the dam for the land preparation of the dry season rice cultivation in 1978 and 1979 has been observed to be around the first week of March (M1). Thus, the prevalent management strategy is inefficient (c.f. (5.1)). This argument can be illustrated by consideration of the hypothetical benefit transformation curve drawn in Figure 2.2 of Chapter 2; for the

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1 Recall that an irrigation policy involves the amount of application which *in our case* is constant over time so that different policies would vary only in the level of that constant.

2 Individual irrigation strategies relate to the timing of the first withdrawal to be considered for the dry season.

M1 strategy the value of *in situ* objective can be increased without any reduction in the level of production of rice. Efficiency can be achieved by moving parallel to the X-axis to the optimal point on the benefit transformation curve which corresponds with advancing the commencement of operations to the last week of February.

To be fair, there can be numerous reasons for this inefficiency in practice. Three obvious possibilities can be listed as follows:

(i) the inability to procure inputs and agricultural credit in time;

(ii) the preference of the community for a longer period of leisure after the harvest of the wet season rice; and

(iii) preoccupation with harvesting operations of *slash and burn* agriculture.

However, an earlier study (Mahendrarajah, 1978) shows that the peak harvesting activities of *slash and burn* agriculture in the village is in January, preceding the harvesting of wet season rice. Alternatively, difficulties in immediate disposal of this rice to obtain cash and other inputs for dry season cultivation has been observed. This has implications for strengthening the agri-support services to ensure availability of credit and other inputs for optimal timing of the dry season's cultivation.

Finally, the leisure preference of the community, if significant, will imply a divergence of the technical optimum we have identified from the economic optimum of the community. In such a situation, a larger leisure utility and hence a delayed commencement of the dry season cultivation has to be compensated by a forfeit of some *in situ* benefits owing to the accompanying reduction of water storage in the dam. An alternative possibility is to reduce the extent of cultivation retaining the same level of *in situ* benefits. The third is a combination of these two. However, leisure preference and related aspects are not pursued in this dissertation.

5.6 Summary of Results

The results presented in this chapter could be summarised as follows:

(i) The minimal supplementary irrigation policy designed ensures at least a minimum application level of 0.5 acre inch per week over a period of 11 weeks. The definition of decision rules were heuristic and drew heavily on the author's field observations.

(ii) For implementing the policy, altogether ten strategies were examined individually in the simulation framework provided in Chapter 4. The simulation model was based on the rainfall and storage data for the year 1976/77 which was also used earlier in the identification of the time series model. The choice of the optimal strategy was aided by three criteria: the amount of withdrawal, mean storage at the end of the dry season and the probability of emptying the dam.

(iii) In order to ascertain the consistency of the results obtained in (ii) above, twenty rainfall years were considered separately. The F4 and F3 strategies turned out to be optimal and second best respectively.

(iv) There appeared to be no advantage in commencing dry season cultivation earlier than the third week of February.

(v) The prevalent practice or strategy of withdrawal, that is, commencing the dry season rice cultivation in the first week of March, is clearly not optimal for the optimality criteria and within the choice space considered here.

CHAPTER 6

SUMMARY AND CONCLUSIONS

The nature of the research involved in this dissertation is summarised in Section 6.1. Then, important implications of the results are presented in Section 6.2. In Section 6.3, some areas for future research are indicated.

6.1 Summary of the Research

This dissertation considered the efficient allocation of water resources for small dam village settlements in the Dry Zone of Sri Lanka. Here an attempt was made to identify the nature of the allocation problem and the appropriate type of optimisation and thence to develop an analytical procedure to find solutions. The approach took into account the dynamic and stochastic nature of the water storage.

This general procedure involved the treatment of the following:

(1) The dual roles of the water resource were explicitly delineated, namely irrigation of rice for the attainment of income and *in situ* community uses. The nature of the conflict between the pursuit of these two objectives for management of the water resource was examined and the need for a trade-off between the two was established.

(2) Consequently, the amount of rice production and the quantity of water storage at the end of the dry season were considered as indices for the level of welfare from the income and *in situ* objectives, respectively. However, the whole range of combinations of these two objectives could not be elicited due to lack of knowledge regarding the actual water-rice production relationship.

(3) Thus the analysis was directed at maximising the season-end storage for a given level of rice production and associated level of water application (irrigation + rainfall).

(4) Two concepts in dealing with water as an input to rice production were reviewed, namely the unique water requirement concept and that of water as a variable input. It was argued that the latter concept was more appropriate to represent water use for rice cultivation at small dam sites. In particular, a moisture stress reduction-yield production function was justified.

(5) Together with the consideration of a fixed intraseasonal water requirement for early maturing rice varieties, this was used to define a neoclassical production function. It related the minimum level of application

of water (rainfall + irrigation) throughout crop growth and the level of yield.

(6) An obvious way to at least *locally*[1] optimise the water resource was seen as matching the cropping calendar with the rainfall distribution in order to make maximum use of the rainfall and hence minimise withdrawal from the dam.

(7) Data from a real dam settlement comprising 30 acres of rice land were used as a case study.

(8) Direct analytical (non-simulation or algebraic) solutions to the problem in (6) are fraught with difficulty due to the nonlinear stochastic and dynamic nature of the water storage, itself influenced mainly by the rainfall which is also stochastic. Consequently, a solution to the problem was sought in a simulation framework. The development of a stochastic simulation model for water storage was based on parameter estimates of a transfer function (TF) time series model relating data on rainfall and water storage, principally.

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1 A local optimum to a given problem is here taken as one which maximises (or minimises) the given objective function within a narrower choice space (framework).

(9) The formulation of the TF model was based on systems analysis concepts.

(10) For the identification of the structure of such a model and the estimation of the parameters therein, recursive time-series analysis was adopted. This was described in Chapter 3. Non-linearities in the system were found and, given the available data length, satisfactorily accounted for by filtering the rainfall. The central algorithms used in the time-series analysis were the instrumental variable - approximate maximum likelihood (IV-AML) which not only yield estimates of the mean of the model parameters but also their normal standard deviation.

(11) Thus the stochastic simulation model was designed to incorporate the uncertainty associated with the model, thereby allowing the representation of storage behaviour in probabilistic terms.

(12) The variations in storage which arise from input variability were highlighted using twenty different years of rainfall data, each in a separate stochastic simulation. The 10 withdrawal strategies were also examined in separate simulations. Thus over 20 by 10, that is 200, simulations were performed.

6.2 Implications of the Research

The results of the research have been summarised in the final sections of Chapters 4 and 5 and will not be repeated here. The aim of this section is to consider two important implications of the results. The first relates to the general applicability of such an analytical approach to water management while the second refers to the value and limitations of our results for management of the specific dam under study.

6.2.1 Applicability of the General Approach

This research reveals that, in the small dam situation, it is possible to develop a simple dynamic water supply model to aid optimal allocation of water among the multifarious uses. Crop production being the only use which involves physical withdrawal of water from such dams, the model can provide a framework for analysis of the different withdrawal policies. Given such a model it is possible to use simulation to assess in a probabilistic sense the effects of different irrigation policies, whilst best achieving the two objectives. The present study, however, was confined to the assessment of different planting time strategies for a dry season rice crop using a single irrigation policy. In a future study, it would be possible, for example, to extend this investigation to find the optimum acreage of rice to plant with the current yield level, in order that a stipulated water level would be retained in the dam at the end of the year. In a similar manner, the approach can also be used to quantify storage levels for various choices of desirable crops other than rice.

The chief merits of the stochastic modelling approach derive from its ability to handle noise corrupted data often obtained by crude measures; and the modest requirement of the data itself. These aspects are particularly valuable in a developing country situation where data are scarce and the cost of procuring them is high.

Another advantage is the potential of the recursive procedure to update the model parameters on receipt of actual rainfall data as the season proceeds. This is useful since, as noted earlier in Chapter 4, the pre-dry season storage simulation model for a year is based on a predicted dry season rainfall. But this storage is best updated as the dry season proceeds.

Obviously, such water storage simulation models can also be developed for many other dam sites. This can be achieved at low cost because the procedure of estimation of the transfer function model can be implemented on a mini-computer. The IV algorithms, as has been pointed out in Chapter 3, can be placed naturally in simple recursive form and hence have low computer storage requirements. In fact, our results suggest a simple model which can even be estimated using a calculator. Certainly, with suitable interface equipment, a mini computer could be used on-line to identify and estimate time-series models for water storage in dams. Stochastic simulation models can then be developed to aid irrigation decisions, assess beforehand the sensitivity of storage for rainfall variability etc. They also have a long term value: apart from the value of such time-series models to make efficient use of water resources, they can also be used generally to assess the effects on storage behaviour of any drastic changes in the catchment such as deforestation, housing etc.

6.2.2 Applicability of the Results for the Case Study

Direct inferences from the results for the case study dam site need to be drawn with caution because the identification and estimation of the model used in the stochastic simulation of water storage is based on only one year's rainfall and storage data. Thus, *the model is not satisfactorily validated at this time*. However, the rainfall for that year used to perform the estimation was clearly representative of an average year and, therefore, the modelling was not performed with extreme data.

It has also been shown in Chapter 5 that the present cropping calendar for the dry season rice cultivation is, in terms of attaining the two objectives, sub-optimal. The possible causes of sub-optimality were examined in Chapter 5. The formulation of the problem also ignored the labour and various resource demands for other components of the total agricultural system. Considerations of such constraints would have narrowed the 'choice space' and, consequently, a different solution would be optimal.

Hence, the implications drawn from the results of this research are pertinent only if the community is not constrained by the above factors. On the positive side, it is highly likely that there is a lack of knowledge in the community about the effects of different times of cultivation. This is evident because this dry season cultivation is new to the community. On the other hand, if the practice is continued, the community will tend to move gradually towards the optimal time of planting after many year's experience. It is in this respect that the simulation approach presented herein is valuable since it may be able to provide the equivalent of many year's experience within minutes.

6.3 Directions for Future Research

The research in this dissertation could be extended in three major areas which can be summarised as follows:

(i) validation of the estimated time-series model and, in particular, further examination of the nature of the non-linearity in the system;

(ii) estimation and incorporation of an accurate water-yield production function; and

(iii) consideration of the statistical distribution of rainfall and other supportive research to enhance the accuracy of prediction of the dry season's rainfall distribution.

The unvalidated nature of the water storage model has been mentioned already. It remains a valuable exercise to obtain more rainfall and storage data so that the structure of the model and its parameter estimates can be confirmed and the constants in the non-linear filters, if appropriate, can be better tuned. For instance, the time constant in the soil moisture filter, T_c, was found to be 6 weeks. This would mean that to account for soil moisture effects a memory of 6 weeks must be considered. This appears to be too large and can, for example, be taken to suggest that the non-linearity may be in the *state* of the system rather than on the input, that is, rainfall.

As mentioned the present investigation considered a given level of yield in a hypothetical water application level-yield production function. The existence of such a production function for early maturing rice varieties was justified in Chapter 2 based on agronomic findings. However, no empirical production function is available. This identifies a useful area of research for irrigation agronomists. Such investigations can also be aided by studies relating to the hydrology of rice land. In particular, the relatively lower water requirement of rice here is worth investigating. A study of ground water level movements in the rice land near small dams, in a manner similar to the one undertaken for the Dry Zone's landscape by Panabokke (1958), will also be valuable. More precise knowledge of the actual water-yield production functions permits a wider choice of production level, yield level and an appropriate extent (acreage). Thus it might be possible to attain the same level of production with a relatively lower acreage of cultivation *rather* than extensive cultivation with a lower level of water application. This may be preferable in times of unforeseen drought where the water application rates on a low acreage - high yielding crop can be reduced without total loss of any of the crop.

A third logical extension to this research is to strengthen the ability to forecast the dry season rainfall pattern based on the wet season's rainfall and/or other considerations. This might be accomplished by undertaking pattern recognition and other statistical studies of rainfall making use of many years rainfall data. It is possible to obtain such data from the closest Meteorological Station. The ability to make accurate predictions of the dry season's rainfall is valuable for more effective pre-season planning of the application rates of water, acreage to cultivate and the time of sowing of the dry season crop.

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APPENDIX A

CASE STUDY DAM SITE: CHARACTERISTICS AND DATA SOURCES

The principal data used in this dissertation were derived from an earlier report by the author with respect to a small dam settlement or village known as Walagambahuwa. This is situated approximately 10 km away from the Dry Zone Agricultural Research Station at Maha illuppallama. Since 1976, it has also gained prominence as a field experiment site of the Sri Lankan Government Department of Agriculture, which continues inter-disciplinary research in this settlement with the view of investigating and developing crop intensification strategies to extend to other small dam situations.

In the first section of this Appendix, some of the general characteristics of Walagambahuwa are given focusing on the dam, rice land and the community. In the second section, details of the rainfall, water level and temperature data used in the analysis are provided while the final section presents the supplementary data on rainfall used in the investigation.

A.1 Walagambahuwa: Characteristics

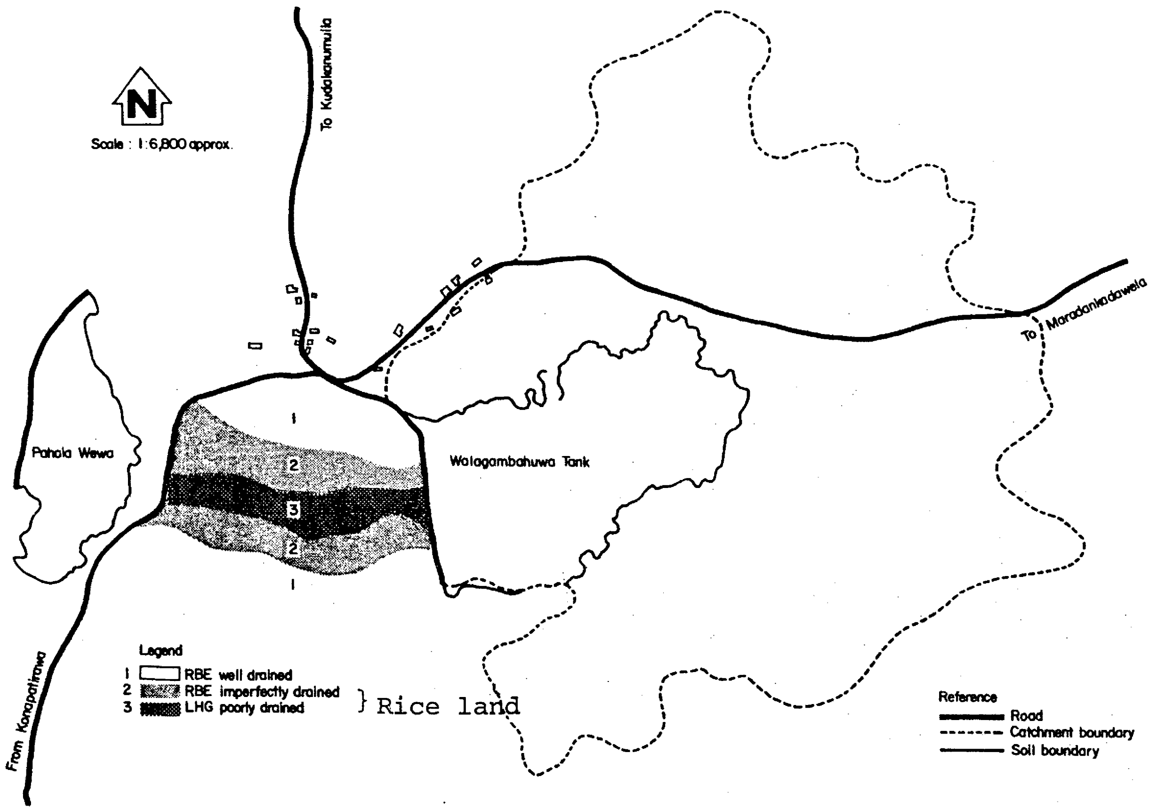
Walagambahuwa is similar in physical and socio-agro-economic conditions to many other villages in the vicinity, although it may not, in any special way, be 'typical' or 'average' of all such villages. Sociological features and prevailing agricultural rituals are also more or less characteristic of traditional small dam settlements in the Dry Zone.

In this village, there are approximately fifty families, mostly kith and kin. In addition to kinship sentiments, economic interdependence, specifically, a joint interest in the rice land and *chena* also unites these families. Almost all of them own, by inheritance, a share in the rice land, which lies immediately below the dam and measures approximately 30 acres in extent. Individual holdings, although small (3/8 to 3/4 acres) in area consist of 2-4 parcels scattered over the block of rice land. Most families own 3 parcels of land: one closer to the dam, the next around the middle and the third in the distal segment of the block. Such a distribution is favourable for *bethma* cultivation whenever necessity arises. In addition to this *paraveni* or *purana* land, a few of the households own recently acquired rice land bordering the former at a relatively higher elevation. This is known as *akkarawela* which is cultivated sometimes during the wet season if the rainfall is very high. In general, the households operate *slash and burn* or *chena* plots ranging in size from 1 to 2 acres. *Home-garden* is a minor component in the agricultural system. As a whole, land distribution in Walagambahuwa is remarkably equitable. This is particularly so in respect of *purana* rice land, which is the concern of our present research, so that equity considerations can be avoided in our land-based efforts towards the improvement of the welfare of the community.

The village dam has a full capacity of approximately 120 acre feet. Until 1976, the Cultivation Committee had responsibility for management of the water resource. Subsequently, it was given over to the research team of the Department of Agriculture in order to facilitate the demonstration of the new management approach in this dam site. The dam, catchment and the rice land of Walagambahuwa are shown in Map A.1.

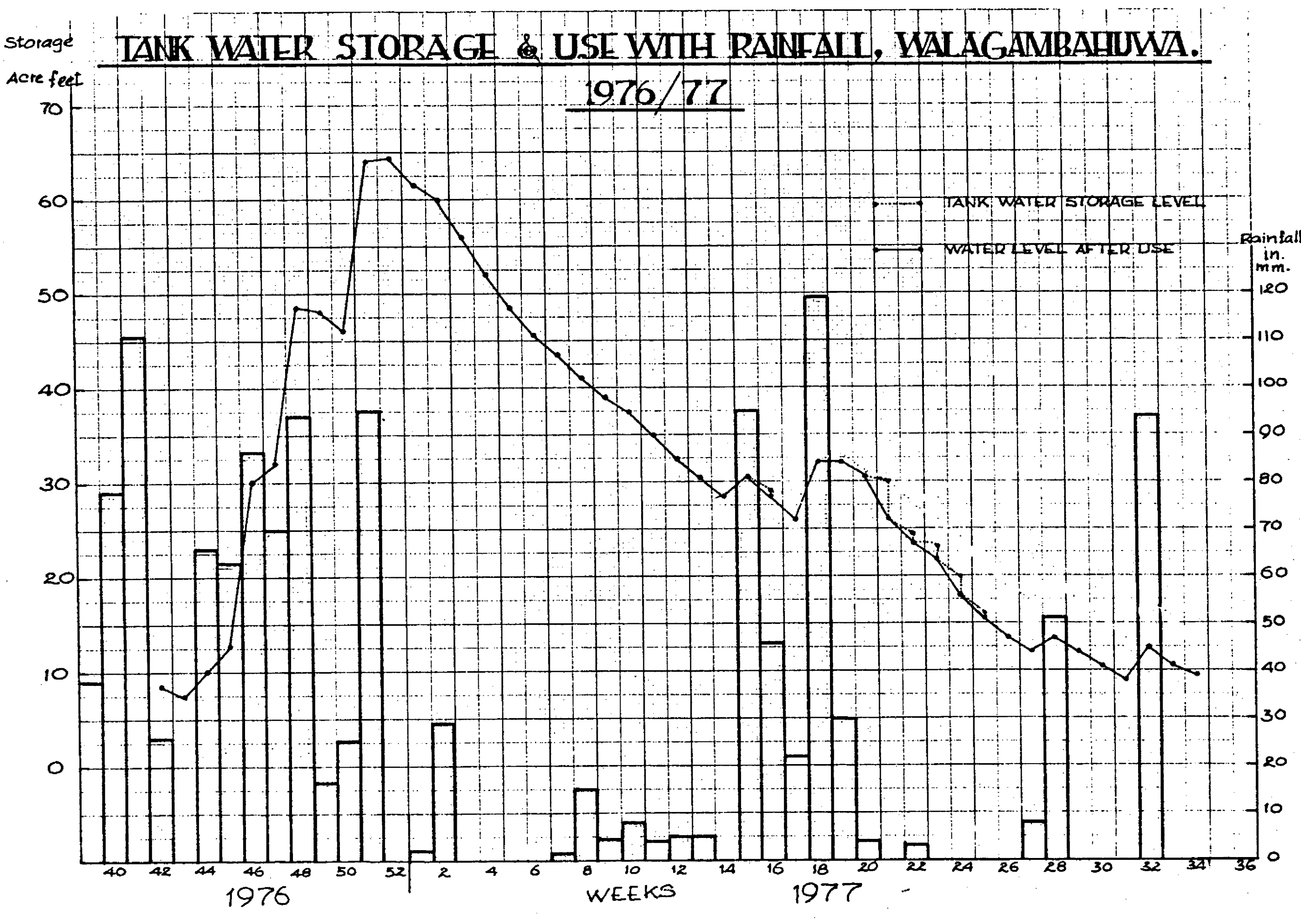
MAP A.1

DAM, CATCHMENT AND RICE LAND OF WALAGAMBAHUWA



Source: Adapted from Somasiri (1976), p.94.

FIGURE A.1



Source: Mahendrarajah (1978, p.28).

A.2 Principal Data

Water storage behaviour and the weekly total rainfall data are provided graphically for this dam by Mahendrarajah (1978) for the period from October 1976 to August 1977. The graph is reproduced here in Figure A.1, which constitutes the major data source for this research. Dam water levels and rainfall data were extracted from this graph. The temperature data used here were also obtained from the same report, which in one of its Appendices provides monthly mean temperatures. Weekly temperatures were obtained by interpolation. The associated errors add to the crudeness of the original measurements.

The monthly mean temperatures reported by Mahendrarajah (1978) had been computed using daily maximum and minimum temperatures recorded for 1976/77 at the Agrometeorological Station at Maha illuppallama. Rainfall data were collected daily at the dam site using a standard rain-gauge. On the other hand, the water storage heights were originally measured daily with a scale cemented on to the dam-floor near the sluice-gate. The water heights were later converted to acre-feet by referring to an 'elevation capacity' calibration curve prepared by the Land Use Division of the Department of Agriculture. The data on water storage level, rainfall and temperature used ultimately in the analysis are presented in Table A.1.

A.3 Supplementary Data

Twenty years rainfall data are presented in Table A.2. They are the actual daily rainfalls recorded at the Agrometeorological Station at Maha illuppallama from September 1959 to

TABLE A.1

WEEKLY TOTAL RAINFALL, WATER LEVEL IN THE DAM AND

MEAN TEMPERATURE, WALAGAMBAHUWA, 1976/77

Week No.	Agricultural Year 1976/77 Period	Data Set Week	Total Rainfall mm	Water Level** Ac.ft	Mean Temperature* F°	Week No.	Agricultural Year 1976/77 Period	Data Set Week	Total Rainfall mm	Water Level** Ac.ft	Mean Temperature* F°
1	2/9 to 8/9 - 1976	-	n.a.	n.a.	n.u.	27	3/3 to 9/3 - 1977	24(21)	8	37.5	79.2
2	9/9 to 15/9 - 1976	-	n.a.	n.a.	n.u.	28	10/3 to 16/3 - 1977	25(22)	4	35.0	80.0
3	16/9 to 22/9 - 1976	-	n.a.	n.a.	n.u.	29	17/3 to 23/3 - 1977	26(23)	5	32.3	80.7
4	23/9 to 29/9 - 1976	1	38	n.a.	83.3	30	24/3 to 30/3 - 1977	27(24)	5	30.5	81.4
5	30/9 to 6/10 - 1976	2	78	n.a.	82.7	31	31/3 to 6/4 - 1977	28(25)	0	28.5	81.9
6	7/10 to 13/10 - 1976	3	111	n.a.	82.2	32	7/4 to 13/4 - 1977	29(26)	95	30.5	82.5
7	14/10 to 20/10 - 1976	4(1)	26	8.5	81.6	33	14/4 to 20/4 - 1977	30(27)	46	29.3	82.9
8	21/10 to 27/10 - 1976	5(2)	0	7.5	81.1	34	21/4 to 27/4 - 1977	31(28)	22	26.8	83.1
9	28/10 to 3/11 - 1976	6(3)	66	10.0	80.5	35	28/4 to 4/5 - 1977	32(29)	119	32.8	83.0
10	4/11 to 10/11 - 1976	7(4)	63	12.8	80.0	36	5/5 to 11/5 - 1977	33(30)	30	32.8	83.0
11	11/11 to 17/11 - 1976	8(5)	86.5	30.0	79.5	37	12/5 to 18/5 - 1977	34(31)	4	31.3	82.9
12	18/11 to 24/11 - 1976	9(6)	70	32.0	78.8	38	19/5 to 25/5 - 1977	35(32)	0	30.8	82.8
13	25/11 to 1/12 - 1976	10(7)	94	48.5	78.6	39	26/5 to 1/6 - 1977	36(33)	3	29.3	82.8
14	2/12 to 8/12 - 1976	11(8)	16.5	48.0	78.4	40	2/6 to 8/6 - 1977	37(34)	0	29.0	82.9
15	9/12 to 15/12 - 1976	12(9)	25	46.0	78.0	41	9/6 to 15/6 - 1977	38(35)	0	27.2	82.9
16	16/12 to 22/12 - 1976	13(10)	95	64.0	77.6	42	16/6 to 22/6 - 1977	39(36)	0	25.2	83.0
17	23/12 to 29/12 - 1976	14(11)	0	64.3	77.3	43	23/6 to 29/6 - 1977	40(37)	0	23.2	83.1
18	30/12 to 5/1 - 1977	15(12)	2.5	61.5	77.1	44	30/6 to 6/7 - 1977	41(38)	8	21.7	83.1
19	6/1 to 12/1 - 1977	16(13)	29	60.0	76.8	45	7/7 to 13/7 - 1977	42(39)	51	23.2	83.0
20	13/1 to 19/1 - 1977	17(14)	0	56.0	76.6	46	14/7 to 20/7 - 1977	43(40)	0	21.7	82.9
21	20/1 to 26/1 - 1977	18(15)	0	52.0	76.5	47	21/7 to 27/7 - 1977	44(41)	0	20.2	82.9
22	27/1 to 2/2 - 1977	19(16)	0	48.5	76.8	48	28/7 to 3/8 - 1977	45(42)	0	18.7	83.0
23	3/2 to 9/2 - 1977	20(17)	0	45.7	77.4	49	4/8 to 10/8 - 1977	46(43)	94	22.0	83.1
24	10/2 to 16/2 - 1977	21(18)	1.5	43.5	77.9	50	11/8 to 17/8 - 1977	47(44)	0	20.4	83.2
25	17/2 to 23/2 - 1977	22(19)	15	41.0	78.5	51	18/8 to 24/8 - 1977	48(45)	0	19.2	83.3
26	24/2 to 2/3 - 1977	23(20)	4.5	39.0	78.8	52	25/8 to 31/8 - 1977	-	n.a.	n.a.	n.u.

Notes: (a)* - Computed from daily records of Maximum and Minimum temperatures in Farenheit. (b)** - Storage level in acre feet on the last day of each week. (c) n.a. and n.u. - denote 'not available' and 'not used' respectively. (d) Figures in parenthesis denote the sequence of weeks with complete data set.

Source: Rainfall and water levels extracted from Figure A.1 above. Temperature figures computed from Mahendrarajah (1978, p.64); originally from records of Agrometeorological Station, Mahailluppallama, 1976-1977.

September 1979.[1] Since May 1977, the rainfall records are in millimetres, whereas they are in inches before. Rainfall is highly variable within short distances. Rainfall records are not available for many years at Walagambahuwa. However, Maha illuppallama is within the same (DL_1) agroecological zone as Walagambahuwa. Therefore, its rainfall can be considered to incorporate and illustrate the effect of a range of possible and realistic rainfall distributions at Walagambahuwa.

1 I am thankful to Mrs Chrisantha Croos for her willing assistance in copying these data from the original records.

TABLE A.2

DAILY RAINFALL RECORD, MAHAILLUPPALLAMA:

2 SEPTEMBER 1959 TO 1 SEPTEMBER 1979

in inches

Year	Day in Fortnight													
59/60	1	2	3	4	5	6	7	8	9	10	11	12	13	14
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.71	1.15	1.30	0.	0.	0.	0.	0.12	0.05	0.06	0.03	0.01	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.41	0.10	0.	1.51	0.58	0.25	0.32	0.	0.30	0.05	0.02	0.40
	1.16	0.	0.54	0.07	0.	0.06	0.	1.49	0.05	0.	0.14	0.92	1.98	0.45
	0.	0.33	0.02	0.19	0.07	0.38	0.	0.69	0.	0.69	0.92	1.06	0.79	0.32
	0.62	1.12	0.	0.26	1.93	1.86	0.69	0.01	0.03	0.	0.	0.	0.01	0.
	0.15	0.71	0.43	2.01	0.22	0.12	0.	0.	0.77	0.02	0.09	0.40	0.	0.
	0.	0.	0.48	0.	0.11	0.23	0.	0.03	0.	0.22	0.93	0.18	0.	0.
	0.	0.05	0.	0.09	0.12	0.	0.	0.07	0.	0.	0.	0.	0.	0.
	1.79	0.03	1.22	0.03	0.36	0.	0.	0.	0.	0.	0.08	0.06	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.29	0.	0.	0.10	0.	0.	0.
	0.	0.	0.56	0.57	0.66	1.54	0.21	0.57	2.93	0.07	0.	0.16	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.43	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.06	0.	0.	0.	0.
	0.18	0.17	0.01	0.	3.87	0.56	0.07	0.	0.71	0.04	0.69	0.	0.	0.
	1.09	3.86	1.00	0.	0.	0.	2.59	0.	0.02	0.88	0.	0.26	0.	0.
	0.31	0.	0.	0.	0.75	0.	0.03	0.14	0.	0.	0.	0.	0.	0.
	0.	0.12	1.58	0.12	0.02	0.	0.67	0.	0.01	0.	0.01	0.	0.01	0.
	0.03	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.30	0.	0.	0.	0.	0.
	0.04	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.05	0.17	0.	0.
	0.07	0.58	0.	0.	0.03	0.15	3.65	0.	0.	0.	0.80	0.44	1.90	0.23
	0.48	0.	0.	0.	0.01	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
60/61	0.	0.	0.	0.	0.	0.	0.06	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	2.71	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.87
	0.	0.	0.	0.	0.	0.31	0.61	0.	0.	0.	0.24	2.70	0.03	0.04
	0.35	0.01	0.	0.75	0.66	0.81	0.98	0.	2.87	0.03	2.26	0.82	0.73	0.04
	0.16	0.15	0.25	0.29	0.58	2.88	0.02	0.	0.	0.	0.21	0.17	1.52	0.04
	0.	0.	0.	0.	0.07	0.58	0.	0.	3.12	1.65	0.07	0.	0.	0.03
	0.19	0.14	0.17	0.	0.	0.	0.	0.	0.	0.02	0.	0.	0.	0.
	0.	0.	0.	0.01	0.39	0.23	0.01	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.52	4.56	0.	0.13	0.08	1.15	1.17	0.76	0.	0.03	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.68	1.10
	0.30	0.35	0.01	0.	0.07	0.86	0.28	0.02	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.28	0.	0.05	0.	0.	0.	0.	0.02
	0.	0.	0.	0.	0.	0.23	0.15	0.	0.	0.52	0.23	0.	0.	0.32
	0.	0.	0.12	0.53	0.08	0.58	0.	0.	0.30	0.	0.	0.	0.	0.41
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.10	0.90	0.59
	0.10	0.	4.62	0.02	0.51	0.	0.23	0.83	0.	0.	0.	0.	0.	0.
	0.	0.	0.30	0.	0.09	0.	0.	0.05	0.	0.02	0.27	0.17	0.	0.
	0.	0.	0.04	0.12	0.	0.	0.	0.	0.	0.02	0.	0.	0.46	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.78	0.69	0.72	0.17	0.01	0.04	0.01	0.52	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.21
	0.39	0.	0.	0.	0.	0.	0.	0.01	0.	0.	0.	0.06	0.	0.

	0.	0.	0.02	0.	0.	0.	0.03	0.	0.	0.	0.06	0.08	0.	0.
61/62	0.	0.	0.	0.	0.	0.	0.	0.01	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.09	0.31	0.	0.	0.	0.	0.
	0.	0.	0.	0.05	0.	0.05	0.	0.	0.	0.23	0.14	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.14
	0.11	0.62	0.	0.	0.	0.62	0.15	0.03	2.43	1.43	0.72	0.	1.72	0.07
	1.47	0.12	0.05	2.44	0.01	0.18	0.08	0.	0.	0.	2.65	0.03	0.	0.70
	0.04	0.25	0.	0.15	0.06	0.13	0.29	0.06	0.17	0.	0.07	0.04	1.56	0.02
	2.13	0.25	0.	0.33	0.21	0.87	0.37	2.88	0.02	0.28	0.	3.15	0.09	0.
	0.02	0.	0.80	0.54	0.	0.06	0.17	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.01	0.22	1.80	0.01	0.	0.	0.	0.	0.13
	1.22	0.45	0.20	0.	0.	0.	0.13	0.43	0.26	0.	0.02	0.	0.	0.
	0.	0.05	0.93	0.69	0.53	0.	0.	0.	0.04	0.	0.	0.	0.13	0.10
	0.03	0.04	0.	0.	0.	0.	0.	0.14	0.	0.03	0.02	0.	0.	0.
	0.	0.22	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.13	0.33	0.75	0.09	0.01	0.	0.	0.	0.	0.	0.	0.
	0.02	0.	0.67	0.41	0.39	0.01	0.07	0.	0.03	0.09	0.23	0.86	0.25	0.05
	0.41	0.81	0.06	0.01	0.15	0.19	0.	0.	0.	0.	1.12	0.37	0.	0.
	0.	0.	0.	0.	0.	0.16	0.	0.	0.85	0.31	0.28	0.	0.	0.01
	0.49	0.	0.98	0.71	0.37	0.09	0.01	0.05	0.53	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.18	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.01	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.02	0.01
	0.36	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.05	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
62/63	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.04	0.07	0.06	0.
	0.31	0.29	0.10	0.05	0.	0.	0.	0.	0.	0.	1.37	0.	0.	0.
	0.	0.04	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.11	0.
	0.05	0.	0.	0.	0.21	0.	0.02	0.77	3.29	0.85	5.86	3.62	0.06	0.31
	0.09	0.	0.38	0.02	3.02	0.20	0.54	0.83	0.27	0.09	0.	0.	0.	0.
	0.06	0.	0.07	0.	0.	0.04	0.13	0.04	0.	0.11	0.56	0.07	0.79	0.20
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.50	0.65	0.	0.	0.10
	0.	0.17	0.84	0.	0.	0.	1.72	0.41	0.07	0.06	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.01	0.	0.10	1.28	0.81	0.36	0.	0.03	1.02
	0.03	0.17	0.89	0.33	2.13	0.05	0.	0.	0.	0.	0.32	0.95	0.66	0.96
	1.96	0.50	0.07	0.19	0.	0.	1.02	0.	0.06	0.	0.	0.	0.	0.18
	0.	0.	0.	0.	0.	0.95	0.13	0.84	0.52	0.	0.	0.02	0.	0.
	0.22	0.44	1.85	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.02	0.02	0.07	0.02	0.	0.	0.	0.	0.43	0.	0.24	0.33
	0.02	0.04	0.	0.	0.	0.	0.	0.	0.58	0.06	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.95	0.03	1.96	0.07	0.	0.68	0.	0.
	0.	0.	0.	0.	0.06	0.88	0.	0.09	0.	0.	0.	0.08	0.	1.95
	0.33	0.	0.	0.64	0.14	0.97	0.	2.69	0.35	2.88	0.	0.	0.	0.
	0.	0.	0.	0.	0.19	0.	0.66	0.	0.	0.	0.	0.	0.	0.
	0.11	0.45	0.	0.14	0.24	0.	0.	0.	0.	0.	0.	0.	0.08	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.13	0.02
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.02
	0.25	0.	0.	0.01	0.22	0.	0.	0.	0.	0.	0.	0.	0.15	0.
	0.05	0.	0.	0.	0.	0.01	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.10	0.12	0.	0.	0.	0.	0.	0.	0.	0.	0.12
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
63/64	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.01	0.	0.	0.	0.	0.03	0.	0.	0.	0.	0.01	0.	0.	0.
	0.	0.14	0.	0.	0.10	1.19	0.10	0.01	0.	0.38	0.21	0.04	0.	0.14
	0.	0.02	0.	0.	0.	0.	0.02	0.	0.33	0.	0.	0.	0.	2.15
	3.17	0.37	0.08	0.88	0.17	0.64	3.02	1.22	0.02	0.24	0.01	0.36	0.	0.
	0.	0.06	0.09	0.	0.19	0.27	0.98	1.57	0.	0.15	0.24	0.	0.	0.45
	3.45	0.	0.79	0.24	0.04	0.20	4.17	1.34	0.10	0.40	0.07	0.21	0.79	0.40
	0.94	0.33	1.53	0.94	0.23	0.05	1.05	0.17	1.09	1.12	0.20	1.16	0.82	5.68

	0.02	0.37	0.07	0.	0.	0.	0.	0.	0.	0.02	0.	0.02	1.19	0.31
	0.	0.73	2.05	0.	0.01	0.45	0.04	0.	1.84	0.23	0.02	0.	0.45	0.84
	0.	0.	0.02	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.47	0.32	0.16	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.20	0.84	0.37	0.	0.	0.	0.	0.	0.01	0.	0.26	0.01	0.05
	0.32	0.	0.	0.	1.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.02	0.62	0.	1.80	0.	0.	0.	0.	0.01	0.07	0.
	0.	0.	0.14	0.	0.39	0.	0.23	0.02	0.21	0.04	0.	0.07	0.23	0.
	0.	0.	0.51	0.	0.	0.	0.	0.	0.01	0.	0.18	0.33	0.	1.32
	0.	0.64	0.03	0.	1.28	1.06	0.01	0.01	0.17	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.01
	0.	0.	0.	0.	0.	0.	0.	0.	0.11	0.05	0.	0.	0.	0.
	0.	0.	0.03	0.	0.	0.	0.	0.	0.	0.	0.	0.04	0.	0.
	0.	0.01	0.	0.02	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.03	0.	0.	0.01	0.67	2.02	0.06	0.01	0.04	0.	0.	0.02
	0.	0.01	0.03	0.07	0.08	0.05	0.	0.	0.	0.	0.	0.	0.	0.
4/65	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.21	0.

	0.	1.74	1.97	0.63	0.	0.02	0.30	0.27	0.06	0.	0.	0.06	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	2.04	0.	0.	0.	0.88	0.09
	0.	0.	0.43	0.	0.	0.	0.	0.	1.05	0.72	0.13	0.	0.03	0.
	0.94	0.08	0.21	0.62	0.05	0.27	0.29	0.18	0.	0.64	0.44	0.16	0.	0.63
	0.17	0.31	0.40	0.13	0.	1.64	0.54	0.48	0.	0.40	0.	0.02	0.65	0.
	0.06	0.	0.	0.	0.	0.	0.07	0.34	0.15	0.	0.88	0.	0.06	0.
	0.	0.	0.03	0.01	0.12	0.08	0.	0.	0.	0.	0.	0.03	0.09	2.25
	0.14	0.	0.06	0.01	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.01	0.	0.06	0.22	0.72	0.91	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.02	0.	0.08	0.04	0.	0.02	0.	0.31
	0.	0.	0.	0.08	0.	0.	0.03	0.	0.	0.32	1.37	0.44	0.22	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.57
	0.06	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.24	0.	0.	0.	0.07	0.09	0.	0.17	0.01	0.97
	2.60	0.	0.	0.	0.	0.07	0.01	0.	0.	0.30	0.	0.	0.	0.05
	0.39	0.	1.05	3.33	0.71	0.03	0.	0.	0.08	0.	0.02	0.	0.01	0.
	0.	0.33	0.	0.07	0.	0.	2.02	0.08	0.20	1.54	0.	0.	0.	0.
	0.	0.	0.	2.85	0.08	0.02	0.	0.	0.	0.	0.01	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.02	0.	0.	0.01
	0.	0.	0.01	0.	0.	0.	0.	0.	0.	0.16	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.07	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.02	0.	0.	0.	0.58
	2.08	0.	0.	0.	0.12	0.32	0.75	0.28	0.09	0.	0.03	2.48	0.50	0.
5/66	0.	0.	0.	0.	0.	0.	0.	0.	0.09	0.02	0.	0.	0.	0.

0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.03	2.27	4.67	0.14	0.97	0.
0.35	0.91	2.90	0.13	0.08	0.07	0.13	0.19	0.87	0.43	0.81	0.30	0.75	0.13
0.	0.	0.	0.32	0.20	0.	1.06	0.42	3.48	0.34	0.	0.	0.93	0.76
1.82	0.03	0.	0.	0.	0.	0.	0.	0.05	0.09	1.18	0.17	1.28	0.19
0.23	0.02	0.27	0.16	0.05	2.44	0.13	4.73	0.20	0.83	1.72	1.05	0.04	0.17
0.03	0.02	0.	0.	0.	0.	0.	0.01	0.	1.05	1.35	0.98	2.92	0.03
0.	0.	0.	0.35	0.04	0.05	0.	0.	0.	0.11	0.	0.28	0.	0.21
0.04	0.02	0.05	0.03	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.15	3.51	0.71	1.77	0.43	0.31	0.	0.
0.	0.	0.	0.	0.	0.	0.03	0.	0.	0.28	0.	0.	0.05	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.35	0.	0.	0.	0.17	0.
0.	0.	0.11	2.03	0.	0.	0.	1.40	0.40	0.	2.72	0.05	0.06	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.02	0.	1.20	0.05
0.	0.	0.	0.	0.	0.	0.08	0.91	0.	0.05	0.	0.25	0.	0.

	0.14	0.20	0.	1.50	0.61	0.	0.	0.08	0.	1.92	0.	0.67	0.02	0.
	0.14	1.51	1.31	0.04	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.01	0.
	0.	0.	0.	0.	0.	0.	0.	0.03	0.	0.06	0.01	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.02	0.	0.	0.05	0.
	0.	0.	0.	0.	0.01	0.	0.	0.	0.03	0.	0.	0.	0.	0.
	0.	0.	0.	0.78	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.13	0.02	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.01	0.	0.	0.03	0.	0.	0.02	1.44
66/67	0.	0.	0.	0.	0.	0.	0.01	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.04	0.08	0.01
	0.	0.	0.	3.12	0.04	0.	0.	0.18	0.	1.22	0.20	0.04	0.70	0.66
	0.	0.	0.01	0.	0.22	0.12	0.24	3.75	0.56	0.19	0.	0.	0.	0.24
	0.07	0.08	1.98	2.01	0.09	0.03	0.10	0.	0.01	0.	0.	0.	0.	0.03
	0.33	1.07	0.62	0.05	0.	0.34	0.01	0.	0.01	0.36	2.13	4.88	1.67	0.32
	0.11	0.44	2.15	0.50	1.72	0.	0.	0.	0.15	0.71	0.	0.18	0.	0.
	0.02	0.	0.	0.02	0.01	0.24	0.60	1.15	0.	0.52	0.13	0.66	0.10	0.43
	0.03	0.	0.	0.	0.	0.	0.14	0.04	0.02	0.	0.	0.16	0.05	0.26
	0.	0.31	0.67	1.33	0.	0.	1.10	1.34	0.01	0.	0.08	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.04	0.12	0.02	0.	0.07	0.05
	0.	0.03	0.04	0.25	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.32	4.25	0.	0.01	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.06	0.06	0.37	0.	0.	0.	0.	0.	0.	0.	0.13
	0.13	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.72	2.98	0.	0.	1.25	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.01	0.	0.	0.63	0.03	0.14	0.
	0.	0.	0.05	0.22	0.	0.	0.	0.63	0.	0.63	0.09	0.	0.	0.06
	0.	0.19	0.	0.36	1.35	0.	0.	0.	0.	0.	0.	0.	0.03	1.14
	0.03	0.	0.	0.02	0.03	0.18	0.21	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.02
	0.06	0.	0.	0.	0.	0.	0.05	0.	0.	0.	0.	0.	0.	0.52
	0.	0.20	0.	0.	0.	0.01	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.21	0.05	0.	0.	0.	0.	0.	0.	0.	0.	0.02	0.
	0.	0.03	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
67/68	0.	0.	0.	0.	0.09	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.32	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.03	0.01	0.01	0.	0.03	0.	0.	0.61	0.25	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.51	0.47	0.10	0.	0.14	1.34	0.58	4.26	0.	0.	0.11	0.	0.	0.
	0.	0.02	4.50	0.11	0.08	0.01	2.38	0.	0.	0.02	0.05	0.	0.	0.01
	1.23	0.	0.	0.	0.01	0.	0.	0.	0.	1.45	0.11	0.10	0.01	0.06
	0.	0.10	1.62	0.26	1.18	0.07	0.14	0.02	0.51	0.33	5.48	5.74	0.09	0.
	0.	0.	0.08	0.12	0.	0.	0.09	0.	0.	0.	0.	0.	0.	0.01
	0.	0.	0.	0.	0.50	0.03	0.	0.	0.	0.01	0.16	0.99	0.	0.
	0.	0.	0.48	0.	0.	0.	0.	0.03	0.	0.	0.	0.	0.	0.
	0.07	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.02	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.01	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.21	0.22	0.37	0.50	0.	2.31	0.15
	0.	0.	0.	0.	0.	0.	0.35	0.84	0.21	0.	0.	0.43	0.	0.
	0.	0.	0.	0.	0.	3.92	0.15	0.15	0.	0.	0.	1.05	0.12	0.50
	0.40	0.	0.	0.	0.	0.	0.14	0.05	0.	2.27	0.	0.	0.06	0.29
	0.09	0.71	0.68	0.	0.	0.	0.03	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.38	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.04
	0.32	0.95	0.02	0.20	0.04	0.	0.01	0.	0.	0.	0.	0.	0.12	0.
	0.02	0.14	0.	0.	0.01	0.	0.	0.	0.	0.	0.	0.	0.01	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.10	0.04	0.11	0.03	0.04	0.
	0.03	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

68/69	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.11	0.64	0.34	0.39	0.	0.	0.	0.	0.	0.	0.10	0.31
	0.04	0.02	0.	0.01	0.	0.	0.	0.	0.	0.	0.	0.	0.07	2.24
	0.06	0.	0.03	0.09	0.13	1.66	0.04	1.28	0.34	0.14	0.11	0.	0.27	0.
	0.04	0.25	2.15	0.66	0.59	0.09	0.57	0.	0.02	0.21	0.01	1.92	1.58	0.
	0.	0.	1.32	0.04	0.19	1.44	0.04	3.82	0.02	0.01	0.10	0.	0.	0.04
	0.06	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	2.94	1.06	0.05
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.02	0.01
	0.20	0.	0.	0.	0.84	0.	0.54	0.09	0.	0.09	0.	0.	0.	0.
	0.16	0.02	0.26	0.10	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.29	0.23	0.26	0.14	0.03	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.64	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.18	0.37
	0.	0.13	0.	0.	2.10	0.	0.	0.	0.	0.	0.	0.	1.95	0.
	0.	0.	0.	0.	0.06	1.29	0.02	0.17	0.	0.18	0.29	0.28	0.16	1.54
	0.	0.	0.	0.	0.	0.	2.48	1.08	0.	0.	0.	0.04	0.	0.41
	0.	0.	0.81	0.	0.21	0.	0.06	0.05	0.	0.	0.	0.	0.19	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.01	0.	0.05	0.	0.	0.	0.	0.	0.02	0.04	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.02	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.04	0.06	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.11	1.55	0.04	0.32	0.60
69/70	0.02	0.02	0.18	0.	0.	0.54	1.63	0.	0.09	0.	0.	0.	0.	0.03
	0.	0.	0.	0.05	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.67	0.	0.
	0.	0.54	0.	0.	0.	2.91	0.32	1.60	0.15	0.	0.25	0.	0.64	0.80
	0.26	0.04	0.28	0.07	0.02	0.94	1.04	0.60	0.47	0.	0.06	0.	0.03	0.
	0.	0.09	0.38	1.28	0.	0.	0.	0.	0.02	0.15	0.	0.	0.	0.13
	0.14	0.08	0.97	0.04	0.10	0.69	0.29	0.	0.	0.01	0.82	0.01	0.	0.
	0.45	0.31	0.	0.15	0.	0.10	0.	0.	0.	0.38	0.	0.	0.20	1.63
	0.36	0.10	0.	0.01	0.	0.	0.	1.10	0.36	0.	0.41	0.31	0.14	0.01
	0.02	0.39	1.85	0.68	0.94	1.44	1.93	1.23	0.29	0.	0.32	0.	0.	0.
	0.67	0.04	0.82	0.24	0.05	0.09	0.03	0.01	0.	0.	0.	0.13	0.55	0.14
	0.29	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.04	0.45	3.57	0.17	0.	0.27	0.07	0.	0.05	0.	0.	0.	0.	0.
	0.22	0.75	0.03	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.06	0.07	0.01	0.	1.87	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.26	0.	0.01	0.58	0.
	0.	0.	0.	0.	0.01	0.72	0.42	0.10	2.74	0.	0.	0.	0.30	0.25
	0.	0.	0.	0.	0.	0.	0.06	0.36	0.	0.14	0.05	0.	0.18	0.02
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.52	0.03	0.59	0.
	0.	0.	0.	0.	0.	2.63	0.01	0.	0.	0.	0.	0.	0.	0.
	0.02	0.	0.	0.70	0.01	0.01	0.03	0.	0.	0.	0.	0.	0.	0.
	0.	0.01	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.04	0.03	0.	0.	0.	0.	0.
	0.	0.	0.42	0.02	0.	0.	0.	0.	0.	0.17	0.01	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.01	0.	0.	0.	0.	0.	0.	0.
70/71	0.	0.	0.	0.	0.	0.01	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.26	0.	1.43	0.	0.	0.	0.
	0.	0.	0.	0.16	0.	0.	0.	0.16	0.	0.	0.	1.08	0.	0.
	0.	0.08	0.	0.	0.	0.	0.	1.15	0.	0.	0.24	0.	0.45	1.30
	0.61	0.14	0.	0.22	0.89	0.93	0.03	0.	0.	0.	0.	0.	0.	0.
	0.20	0.	0.04	0.14	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.01	0.66	1.68	0.10
	0.43	0.46	0.12	0.61	0.99	0.02	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.01	0.29	0.08	0.60	0.	1.14	0.72	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.29	0.	0.71	0.03	0.01	0.	0.03	2.23	0.02	0.10	0.
	0.	0.05	0.	0.	0.	0.03	0.	0.46	0.	0.	0.	0.27	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.22	0.	0.	1.17	0.20	0.	0.20	0.	0.	0.	0.	0.35	1.12
	0.	0.05	0.	0.17	1.37	0.07	0.16	0.	0.13	0.19	0.12	0.20	0.40	0.
	2.25	0.38	2.84	2.25	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.68	0.12	0.75	0.02	0.	0.	0.	0.	0.
	0.	0.02	0.	0.	0.	0.	0.	0.	0.	0.	0.02	0.07	0.	0.
	0.02	0.02	0.	0.	0.	0.	0.	0.	0.	0.16	0.85	0.16	0.09	0.10
	0.	0.06	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.06	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.01	0.01	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.07	0.08	0.	0.	0.	0.08	0.75
71/72	0.89	2.31	0.10	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.32	0.	0.	0.	0.	0.
	0.93	0.12	0.	0.	0.	0.22	0.30	0.09	0.46	0.09	0.02	0.	0.08	0.29
	0.01	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.66	1.19	0.75	0.29	0.40	0.48	0.59	0.67	0.03	1.18
	0.11	1.15	0.	0.	0.01	0.80	0.65	0.84	0.	0.	0.21	0.	0.	0.
	0.	0.	0.	0.	1.57	0.37	0.	0.	0.	0.02	0.02	0.	0.	0.09
	0.04	0.14	0.	0.	0.10	0.03	0.	0.08	0.27	1.24	0.70	0.30	1.72	2.29
	8.85	2.26	0.11	0.97	0.15	1.34	0.30	1.40	0.22	0.05	0.	0.	0.	0.
	0.11	0.	0.04	0.	0.32	0.	0.08	0.33	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.02	0.07	0.40
	0.01	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.48	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.46	0.	0.	0.	0.	0.	0.	0.	0.	0.07
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.17	0.	0.29	0.	0.	0.22	0.	0.02	0.01	0.40	0.02	1.34	0.
	0.	0.	0.05	0.57	0.07	1.56	0.08	0.29	0.29	0.04	0.	0.02	0.25	0.
	0.	0.57	0.30	0.37	0.10	0.	0.15	0.33	0.02	0.	1.09	0.09	0.92	0.26
	0.42	0.99	0.45	0.67	0.02	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.04	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.75	0.	0.	0.	0.	0.02	0.	0.	0.
	0.	0.04	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
72/73	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.29	0.
	0.	0.	0.	0.	0.	0.02	0.12	0.43	0.45	0.	0.	0.19	1.61	0.08
	0.	0.07	0.50	2.22	0.06	0.10	0.08	0.	0.	0.	0.04	0.73	0.10	1.38
	0.42	0.82	0.18	2.08	1.25	0.	0.28	0.15	0.02	0.	0.26	1.05	0.06	0.06
	0.01	0.67	0.16	0.	0.	2.47	0.55	0.13	0.	0.	0.07	0.25	0.15	1.62
	0.16	0.	0.	0.06	0.11	0.96	0.02	0.22	0.	0.	0.	0.	0.	0.
	0.	0.04	0.61	0.02	0.15	0.	0.	0.	0.25	0.45	0.	0.	0.	0.
	0.15	0.	0.	0.	0.03	0.	0.27	0.20	0.35	0.07	0.20	2.21	0.12	0.01
	0.11	0.16	1.23	0.30	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.18	0.	0.	0.	0.	0.
	0.	0.	0.03	0.	0.	0.	0.	0.34	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.12	0.02	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	1.40	1.71	0.06	0.	0.	0.	0.	0.	0.05
	0.35	0.	0.	0.77	0.	0.	0.	0.	0.23	1.20	0.	0.32	0.	0.
	0.	1.04	0.07	0.	0.05	0.	0.	0.	0.04	0.	0.	0.05	0.	0.

	0.	0.	0.	0.	0.24	0.56	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.01	0.	0.21	0.30	0.	0.	0.
	0.11	0.	0.	0.	0.	0.	0.	0.	0.	0.06	0.	0.	0.	0.
	0.	0.	0.07	0.	0.	0.	0.	0.01	0.02	0.08	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.04	0.	0.	0.05	0.43	0.40
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.03	1.08	0.	0.	0.	0.	0.01	0.02	0.	0.05	0.02	0.	0.	0.
	0.	0.	0.	0.	0.	0.68	0.03	0.42	0.	0.	0.	0.	0.	0.
73/74	0.	0.	0.	0.	0.	0.	0.	0.	0.07	0.02	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.10	2.43	0.83
	0.	0.	0.12	0.	0.	0.	0.	0.	0.	0.12	0.	0.90	0.10	0.
	0.	0.06	0.	0.	0.	0.	0.	0.	0.	0.	0.09	0.	0.	0.
	3.67	0.	0.15	0.10	0.17	0.19	0.15	0.02	0.73	0.07	0.08	0.19	0.59	1.11
	0.05	1.40	0.23	0.10	0.	0.02	0.46	0.17	0.24	0.10	0.	0.02	0.	0.
	0.	0.	0.	0.	0.	0.01	0.	0.01	0.	0.	0.	0.	0.	0.61
	0.05	0.01	0.03	0.05	0.06	0.03	0.63	0.07	0.20	0.53	0.17	0.	0.	0.
	0.	0.	0.	0.47	0.09	0.64	0.13	0.04	0.42	0.65	0.	0.04	0.02	0.
	0.62	1.32	0.28	1.94	3.83	1.13	0.	1.85	0.	0.	0.	0.	0.	0.
	0.	0.02	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.08	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	2.14	0.	0.87	0.76	0.12	0.48
	0.	0.	0.	0.	0.	0.	0.	0.	0.03	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.19	0.	0.12	0.	0.	0.07
	0.49	0.14	0.	0.	0.68	0.09	0.04	0.	0.16	0.95	1.36	0.03	1.72	0.
	0.	0.11	2.87	0.	0.	0.	0.	0.	2.11	0.	0.	0.17	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.01	0.	0.	0.	0.	0.
	0.	0.05	0.	0.17	0.	0.02	0.	1.59	0.	0.01	0.01	0.09	0.23	0.
	0.05	0.42	0.10	0.02	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.03
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.17	0.16	0.	0.	0.01	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.01	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.23	0.	0.	0.	0.03	0.01	0.
74/75	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.03	0.	0.03	5.86	1.15	0.07
	0.07	0.13	0.	0.	0.	0.08	0.53	0.03	0.	0.	0.	0.	0.	0.
	0.06	0.	0.	0.	0.	0.	0.	0.01	0.03	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.76	0.
	0.	0.34	0.	0.	0.	0.	0.	0.01	0.	0.	0.34	0.66	0.	0.
	0.	0.	0.	0.	0.05	0.	0.	0.	0.	0.	0.	0.35	0.57	0.58
	1.30	0.	0.	0.	0.03	0.	0.	0.	0.	0.	0.34	0.02	0.	0.31
	0.02	1.06	1.95	0.05	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.02
	0.06	0.09	0.66	0.05	0.19	1.97	0.04	0.	0.	0.	0.03	0.	0.01	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.04	0.	0.55	0.36
	0.25	0.	0.	0.05	0.56	0.50	0.03	0.	0.	0.	0.	0.	0.	0.
	0.17	0.76	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.01	0.06	0.	0.	0.	0.	0.04	0.44	0.16
	0.64	0.50	1.29	0.90	0.33	0.	0.	0.	0.28	0.	0.	0.11	0.	0.
	0.	0.	0.	0.01	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.74	0.	0.	0.	1.70	0.69	0.13	0.	0.08	1.10	0.41	0.
	0.	0.	1.26	0.	0.	0.	0.07	0.	0.14	0.15	0.18	0.	0.01	0.85
	2.87	0.04	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	5.11	0.
	0.02	0.	0.08	0.	0.	0.	0.	0.	0.	0.10	0.44	0.03	0.	0.
	0.	0.	0.	0.	0.	0.	0.02	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.01	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.17	0.	0.	0.	0.	0.	0.	0.04	0.07	1.64
	0.	1.00	0.13	0.33	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.05	0.01	0.	0.	0.	0.	0.	0.
75/76	0.	0.	1.23	0.02	0.	0.	0.	1.18	0.	0.	0.	0.	0.	0.05

	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.01
	0.48	0.	0.11	0.01	0.	0.03	1.05	0.	0.	0.02	0.24	0.	0.	0.
	0.	0.	0.	0.	0.63	0.	0.	0.	0.	0.04	0.	0.	0.	1.00
	0.02	0.	0.	0.	0.05	0.	0.	0.85	0.17	0.22	1.09	0.43	0.06	0.07
	0.05	0.21	0.22	0.03	0.30	0.08	0.04	0.	0.01	0.04	0.14	1.22	0.15	0.09
	0.04	0.01	0.17	0.03	0.12	0.78	2.04	0.59	0.04	0.98	0.20	0.30	0.43	0.07
	0.09	0.16	0.03	0.	0.	0.	0.	0.	0.	0.06	0.15	1.24	0.92	0.
	0.	0.	0.	0.11	0.	0.68	0.45	0.35	0.25	0.46	0.50	0.	0.	0.12
	0.11	0.	0.	0.	0.	0.13	0.05	0.	0.	0.	0.	0.	0.	0.04
	0.	0.	0.	0.08	0.03	0.	0.01	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.09	0.37	0.02	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.60	0.46
	0.09	1.78	1.57	1.99	0.37	0.	0.	0.	0.04	0.15	0.83	0.	1.72	0.
	0.	1.20	0.34	0.43	0.08	0.	0.	0.	0.15	1.00	0.	0.21	0.87	0.
	0.15	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.01	0.	0.02	0.
	0.	0.	0.	0.	0.	0.	0.	0.01	0.	0.10	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
6/77	0.	0.	0.	0.	0.	0.	0.01	0.01	0.	0.03	0.04	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	1.78	0.04	0.	0.34	0.	0.	0.06	0.	4.62	0.48	0.	0.37	0.15	0.55
	1.11	1.69	0.	0.33	0.	0.02	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.14	0.	1.25	0.97	0.47	0.09	0.	0.06	1.46	0.06	0.	0.03	1.62
	0.14	1.31	0.02	0.36	0.05	0.10	0.21	0.18	0.63	0.01	0.29	0.01	0.42	0.26
	0.93	0.10	0.42	0.24	0.04	1.94	0.88	0.16	0.06	0.31	0.11	0.33	0.	0.
	0.11	0.	0.01	0.66	0.	0.02	0.	0.55	1.34	0.	0.01	1.10	1.99	0.03
	0.02	0.27	0.61	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.20
	0.	0.	0.10	3.17	0.07	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.23	0.	0.	0.	0.	0.24	0.	0.	0.	0.	0.	0.	0.	0.
	0.15	0.06	0.59	0.	0.	0.27	0.	0.	0.	0.	0.	0.	0.22	0.05
	0.	0.	0.03	0.37	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.02
	0.	0.46	0.06	0.14	0.	0.	0.	0.	0.20	0.	0.	0.	0.01	0.
	0.	0.	0.	0.	0.	0.	1.23	0.	0.	0.11	0.58	0.	0.04	0.06
	0.	0.	0.80	0.40	0.04	0.03	0.37	0.22	0.07	0.22	0.09	0.07	0.06	0.05

in millimetres

	49.5	30.2	0.0	4.8	27.2	4.1	54.4	0.8	1.5	0.5	12.4	0.8	3.8	0.0
	0.	0.	0.	0.	0.	0.	0.	0.4	0.	0.	0.	0.	0.	0.
	0.	0.	0.	1.1	3.6	2.3	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	1.6	0.	0.	0.	0.	0.	0.	0.	0.8	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.8
	0.	0.	0.	1.5	4.5	2.9	19.3	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	97.2	0.	0.	0.	0.	0.	0.
77/78	4.3	0.	1.8	2.0	0.	0.	0.	3.1	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	66.3	5.0	0.7	0.	1.5	0.
	18.5	17.7	0.	0.1	4.0	16.7	17.6	0.	3.5	18.9	8.4	0.	31.4	1.2

	49.0	9.3	2.1	59.6	34.6	2.8	53.8	22.3	79.3	42.8	21.1	3.0	0.	0.
	25.8	0.	0.	0.	0.	0.	14.8	7.4	1.1	1.0	17.4	2.4	0.	0.
	20.7	7.4	146.8	1.9	1.3	0.	0.	0.	0.	0.	0.	1.5	1.5	10.4
	30.0	7.4	0.	0.	0.	0.	8.6	0.3	1.0	0.	0.	0.	0.	0.
	0.	1.7	0.8	26.9	0.5	0.	0.	6.0	48.9	6.5	1.5	24.6	1.1	2.5
	7.7	1.0	0.	0.	0.	0.	0.	0.	3.6	0.	3.6	7.0	0.	0.
	0.	0.	0.	0.	0.	0.	2.0	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	16.8	0.6	0.4	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	43.4	0.	0.	0.	0.	0.	0.	0.	0.	0.	25.2	0.
	0.	0.4	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.2	2.0	0.	15.0	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	41.0	0.	0.	0.	0.	4.1	0.	0.
	0.	0.	0.	0.	0.	0.	18.2	7.3	0.	0.6	0.4	0.	0.	0.
	26.6	0.	0.	2.6	36.3	0.	4.9	40.3	0.	15.7	0.	0.	73.3	1.0
	0.	4.8	8.1	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.3	0.	0.	1.3	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.1	14.0	96.3	0.5	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.5	0.	0.	0.	0.	0.	0.	0.
78/79	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	18.6	0.	0.2	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.2	2.2	0.	0.
	5.9	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.2	0.	0.	0.
	0.	0.	1.9	3.3	0.6	13.3	18.5	60.9	53.6	0.1	5.0	2.1	31.7	1.6
	94.8	1.4	9.7	0.	20.8	93.9	17.5	39.1	2.9	1.6	0.	2.3	2.4	7.2
	5.6	0.	0.	0.	0.	0.	0.	0.	0.	4.7	0.8	26.7	28.6	39.0
	0.	0.	0.	3.5	0.	0.	0.	0.	0.5	3.3	12.0	0.	4.5	5.2
	8.5	16.1	15.0	7.9	1.5	0.	0.	3.3	0.	0.	0.	0.	0.	9.4
	110.6	3.5	18.8	3.8	0.	1.7	0.	0.	0.3	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	3.2	5.9	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	21.3	0.	1.6	0.	0.	0.	0.	0.	0.	0.	0.4
	0.	3.5	1.6	0.	0.	0.	8.3	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.8
	0.	0.	0.	0.	0.	0.	0.	0.	0.	19.0	0.	0.	0.	21.3
	15.9	7.4	3.8	14.0	30.2	3.8	0.	0.	0.	0.	0.	0.4	0.	0.
	0.	0.	0.	0.	0.	3.9	0.	0.	0.	0.	0.	23.8	0.	0.
	0.2	0.	0.	0.	0.	1.4	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.4	0.	2.2	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	1.1	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.5	0.3	0.	0.	0.
	0.	0.	0.	15.6	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.2	3.6	0.

Source: SRI LANKA, Department of Meteorology, Unpublished Weather Records: 1959-1979, Agrometeorological Station, Mahailluppallama.

APPENDIX B

CAPTAIN – INTERACTIVE MODELLING

EXHIBIT B.1 : PRELIMINARY MODEL STRUCTURE IDENTIFICATION

```
YOU WANT? ITERIV, TVAR, AML, PREPROCESS DATA, GRAPHS,  FINISH, XPLANATION
OR HELP. ENTER I,T,A,P,G,F,X,H.
>I
ENTER TITLE FOR PLOTS - UP TO 36 CHARS
>MODEL STRUCTURE IDENTIFICATION
ENTER NO. OF POINTS PER INCH OF PLOT
>6
YOU HAVE   45 DATA POINTS
YOUR DATA PLOTS WILL EACH NEED    1 PLOT PAGES
COMPUTER CHOOSE BEST MODEL? ENTER Y,N,D OR X
>D
ENTER NUMBER OF ITERATIONS REQD
>8
ENTER A AND B VALS, OR E FOR END, OR X
>A1
>A2
>B0
>B1
>B2
>E
A(1)=   .00000    B(0)=   .00000
A(2)=   .00000    B(1)=   .00000
                  B(2)=   .00000
NO A'S= 0 NO B'S= 1 LOG EVN=-.51581+01 R2=-.36011+01 PCT NEVN= .12701+01
NO A'S= 0 NO B'S= 2 LOG EVN=-.48042+01 R2=-.28375+01 PCT NEVN= .26114+01
NO A'S= 0 NO B'S= 3 LOG EVN=-.47464+01 R2=-.22796+01 PCT NEVN= .33696+01
NO A'S= 1 NO B'S= 1 LOG EVN=-.61713+01 R2= .50692+00 PCT NEVN= .95697+00
NO A'S= 1 NO B'S= 2 LOG EVN=-.61410+01 R2= .56101+00 PCT NEVN= .57105+01
NO A'S= 1 NO B'S= 3 LOG EVN=-.60861+01 R2= .54256+00 PCT NEVN= .15259+02
NO A'S= 2 NO B'S= 1 LOG EVN=-.22900+01 R2= .58228+00 PCT NEVN= .50897+02
INSTABILITY, NO.A=2 NO.B=2 ITER=  4
NO A'S= 2 NO B'S= 2 LOG EVN= .99999+03 R2= .00000    PCT NEVN= .99999+03
INSTABILITY, NO.A=2 NO.B=3 ITER=  3
NO A'S= 2 NO B'S= 3 LOG EVN= .99999+03 R2= .00000    PCT NEVN= .99999+03
ENTER A'S AND B'S IN CHOSEN SOLUTION
>1,1
ITERATION   1
A(1)=  -.94286+00 B(0)=   .10004+00
ITERATION   2
A(1)=  -.94093+00 B(0)=   .97221-01
ITERATION   3
A(1)=  -.94158+00 B(0)=   .96580-01
ITERATION   4
A(1)=  -.94149+00 B(0)=   .96470-01
ITERATION   5
A(1)=  -.94152+00 B(0)=   .96419-01
ITERATION   6
A(1)=  -.94152+00 B(0)=   .96400-01
ITERATION   7
A(1)=  -.94152+00 B(0)=   .96391-01
ITERATION   8
PARAM    FINAL VALUE  ST.ERROR     P MATRIX
A(1):  -.94152+00   .50290-01   .25046-04
B(0):   .96391-01   .40595-01   .16320-04
R2= .5069+00 LOG(EVN)=-.6171+01 PCT(N EVN)= .9570+00
SUMMARY OF INPUT: MODEL STRUCTURE IDENTIFICATION
COMP.CHSE=D   L.SQ.REQD=Y   ITERNS=   8   MOD.ORD=  3
```

EXHIBIT B.2 : MODEL STRUCTURE IDENTIFICATION AND BASIC IV-AML ESTIMATION

```
YOU WANT?  ITERIV, TVAR, AML, PREPROCESS DATA, GRAPHS, FINISH, XPLANATION

OR HELP.  ENTER I,T,A,P,G,F,X,H.
>I
ENTER TITLE FOR PLOTS  - UP TO 36 CHARS
>MAHEN
ENTER NO. OF POINTS PER INCH OF PLOT
>6
YOU HAVE   45 DATA POINTS
YOUR DATA PLOTS WILL EACH NEED   1 PLOT PAGES
COMPUTER CHOOSE BEST MODEL?  ENTER Y,N,D OR X
>D
ENTER NUMBER OF ITERATIONS REQD
>8
ENTER A AND B VALS, OR E FOR END, OR X
>A 1
>A 2
>B 0
>B 1
>B 2
>B 3
>A 3
>E
A(1)=    .00000    B(0)=    .00000
A(2)=    .00000    B(1)=    .00000
A(3)=    .00000    B(2)=    .00000
                   B(3)=    .00000
NO A'S= 0 NO B'S= 1 LOG EVN=-.19938+01 R2=-.37408+01 PCT NEVN= .57389+01
NO A'S= 0 NO B'S= 2 LOG EVN=-.16941+01 R2=-.30221+01 PCT NEVN= .11352+02
NO A'S= 0 NO B'S= 3 LOG EVN=-.16553+01 R2=-.25141+01 PCT NEVN= .15155+02
WARNING:  2 NEGATIVE PHAT VALUES
NO A'S= 0 NO B'S= 4 LOG EVN=-.37161+01 R2=-.21424+01 PCT NEVN= .21464+17
NO A'S= 1 NO B'S= 1 LOG EVN=-.71002+01 R2= .97931+00 PCT NEVN= .12396+00
NO A'S= 1 NO B'S= 2 LOG EVN=-.65412+01 R2= .96938+00 PCT NEVN= .57088+01
NO A'S= 1.NO B's= 3 LOG EVN=-.61119+01 R2= .93513+00 PCT NEVN= .35306+01
INSTABILITY, NO.A=1 NO.B=4 ITER=  2
NO A's= 1 NO B'S= 4 LOG EVN= .99999+03 R2= .00000    PCT NEVN= .99999+03
NO A'S= 2 NO B'S= 1 LOG EVN=-.55843+01 R2= .96719+00 PCT NEVN= .10412+02
WARNING:  3 NEGATIVE PHAT VALUES
NO A'S= 2 NO B's= 2 LOG EVN= .33000+02 R2= .97922+00 PCT NEVN= .21464+17
INSTABILITY, NO.A=2 NO.B=3 ITER=  3
NO A'S= 2 MP B'S= 3 LOG EVN= .99999+03 R2= .00000    PCT NEVN= .99999+03
INSTABILITY, NO.A=2 NO.B=4 ITER=  2
NO A'S= 2 NO B'S= 4 LOG EVN= .99999+03 R2= .00000    PCT NEVN= .99999+03
NO A'S= 3 NO B'S= 1 LOG EVN=-.52232+01 R2= .97060+00 PCT NEVN= .25182+01
INSTABILITY, NO.A=3 NO.B=2 ITER=  3
NO A'S= 3 NO B'S= 2 LOG EVN= .99999+03 R2= .00000    PCT NEVN= .99999+03
INSTABILITY, NO.A=3 NO.B=3 ITER=  5
NO A'S= 3 NO B'S= 3 LOG EVN= .99999+03 R2= .00000    PCT NEVN= .99999+03
INSTABILITY, NO.A=3 NO.B=4 ITER=  2
NO A'S= 3 NO B'S= 4 LOG EVN= .99999+03 R2= .00000    PCT NEVN= .99999+03
ENTER A'S AND B'S IN CHOSEN SOLUTION
>1,1
```

SYSTEM MODEL ESTIMATION - BASIC IV

```
A(1)=   .00000    B(0)=   .00000
ITERATION   1
A(1)=  -.93901+00 B(0)=   .65762+00
ITERATION   2
A(1)=  -.93989+00 B(0)=   .64271+00
ITERATION   3
A(1)=  -.93996+00 B(0)=   .64093+00
ITERATION   4
A(1)=  -.93997+00 B(0)=   .64055+00
ITERATION   5
A(1)=  -.93998+00 B(0)=   .64044+00
ITERATION   6
A(1)=  -.93998+00 B(0)=   .64040+00
ITERATION   7
A(1)=  -.93998+00 B(0)=   .64039+00
ITERATION   8
PARAM    FINAL VALUE  ST.ERROR     P MATRIX
A(1):  -.93998+00   .83323-02   .19789-04
B(0):   .64039+00   .39753-01   .45045-03
R2= .9793+00 LOG(EVN)=-.7100+01 LOG(N EVN)=-.6692+01
SUMMARY OF INPUT: BASIC IV-AML ESTIMATION
COMP.CHSE=Y   L.SQ.REQD=Y   ITERNS=   8   MOD.ORD=  1
```

NOISE MODEL ESTIMATION - BASIC AML

```
YOU WANT? ITERIV, TVAR, AML, PREPROCESS DATA, GRAPHS,  FINISH, XPLANATION
>A
ENTER NO OF ITERATIONS
>8
ARE MEANS TO BE SUBTRACTED FROM SERIES? ENTER Y OR N
>N
DATA FROM INPUT DATA OR RESIDUAL SERIES? ENTER I OR R
>R
VARIANCE RESIDUAL=  .350796+01
ENTER C AND D VALS, OR E FOR END
>C 1 0.
>E
C(1)=   .00000
ITERATION NO: 1
C(1)=  -.56129+00
ITERATION NO: 2
C(1)=  -.56183+00
ITERATION NO: 3
C(1)=  -.56190+00
ITERATION NO: 4
C(1)=  -.56192+00
ITERATION NO: 5
C(1)=  -.56193+00
ITERATION NO: 6
C(1)=  -.56193+00
ITERATION NO: 7
C(1)=  -.56193+00
ITERATION NO: 8
C(1)=  -.56129+00
VARIANCE EHAT=  .292434+01
MEANS.SUB=N   ITERNS=   8   MOD.ORD=  1   INPUT.SER=R

INDEX   C VALUE     C ST.ERR    D VALUE     D ST.ERR
  1   -.56193+00   .12456+00
AKAIKES AIC(1) =    .4927677+02
```

EXHIBIT B.3 : 'REFINED' IV-AML ESTIMATION

```
 TYPE MODEL ORDERS (SYSTEM THEN NOISE), TIME DELAY
 ITER IN 4I4 FORMAT
>   1   1   0   9
 ARE LEAST SQUARES REQUIRED ON FIRST ITERATION
>Y
 PROCESS MODEL
          SYSTEM MODEL ORDER = 1  TIME DELAY = 0  # OF SAMPLES = 45
 NOISE MODEL ORDER = 1
 TYPE IN A AND B PARAMETERS AND INITIAL VALUES
 FORMAT (A1,I2,F12.6), E TO END, R TO REMOVE MISTAKE
>A 1
 A(1) =   .000000
>B 0
 B(0) =   .000000
>E
 # PARAMETERS = 2
 TYPE IN C AND D PARAMETERS AND INITIAL VALUES
 FORMAT (A1,I2,F12.6), E TO END, R TO REMOVE MISTAKE
>C 1
 C(1) =   .000000
>E
       PROCESS PARAMETERS                 AUXILIARY MODEL PARAMETERS
 DENOMINATOR        NUMERATOR         DENOMINATOR            NUMERATOR
 INITIAL VALUES :
 A(1) =    .0000    B(0) =  .0000      ALPHA(1)       .000    BETA(0) = .000

       NOISE MODEL PARAMETERS
 DENOMINATOR         NUMERATOR
 INITIAL VALUES :
 C(1) =    .0000000000000  D(1) =   .0000000000000

 ITERATION NUMBER : 1
 A(1) =    -.9390    B(0) =    .6575   ALPHA(1) =     .000    BETA(0)    .000
 C(1) =    -.6542306082934    D(1) =    .0000000000000

 ITERATION NUMBER : 2
 A(1) =    -.9357    B(0) =    .7136   ALPHA(1) =    -.939   BETA(0) = .658
 C(1) =    -.6384325884812    D(1) =    .0000000000000

 ITERATION NUMBER : 3
 A(1) =    -.9378    B(0) =    .6882   ALPHA(1) =    -.936   BETA(0) = .714
 C(1) =    -.6059049841568    D(1) =    .0000000000000

 ITERATION NUMBER : 4
 A(1) =    -.9374    B(0) =    .6938   ALPHA(1) =    -.938   BETA(0) = .688
 C(1)      -.5979776278241    D(1) =    .0000000000000

 ITERATION NUMBER : 5
 A(1) =    -.9377    B(0) =    .6903   ALPHA(1) =    -.937   BETA(0) = .694
 C(1) =    -.5929559410635    D(1) =    .0000000000000

 ITERATION NUMBER : 6
 A(1) =    -.9378    B(0) =    .6893   ALPHA(1) =    -.938   BETA(0) = .690
 C(1) =    -.5901600521523    D(1) =    .0000000000000

 ITERATION NUMBER : 7
 A(1) =    -.9379    B(0) =    .6880   ALPHA(1) =    -.938   BETA(0) = .689
 C(1)      -.5883244609006    D(1) =    .0000000000000

 ITERATION NUMBER : 8
 A(1) =    -.9380    B(0) =    .6870   ALPHA(1) =    -938    BETA(0) = .688
 C(1) =    -.5870587196233    D(1) =    .0000000000000
```

```
SIGMA**2 =          3.8711497722363
MEAN OF PSIHATS =           .3725364464699
VARIANCE OF PSIHATS =     5.6368680070932

TOTAL CORRELATION COEFFICIENT (RT**2)
AROUND MEAN OF OUTPUT DATA =  .9721712

P MATRIX DIAGONALS
  2.03472-006
  1.24916-004

                    SYSTEM COVARIANCE MATRIX
          .0000078767036      .0000498250311
          .0000498250311      .0004835684322

LN(EVN)  =  -8.31131
LN(NEVN) =  -7.94018

NOISE COVARIANCE MATRIX
          .0072841737521      .0000000000000
          .0000000000000      .0000000000000
```

APPENDIX C

COMPUTER PROGRAMS

```
C   PROGRAM 1:
C   ---------
C
C     PURPOSE: TO FILTER THE RAINFALL DATA FOR 'SOIL' AND
C              ATMOSPHERIC EFFECTS IN ORDER TO LINEARISE
C              THE IMPULSE RESPONSE OF RAINFALL-STORAGE
C
C     INPUTS:  WEEKLY MEASURED RAINFALL, U(K)
C              WEEKLY AVERAGE TEMPERATURE,T(K)
C              BOTH FOR 48 WEEKS
C
C     PROCESS: 1) TEMPERATURE FILTER MODEL
C                    UT*(K)  = C(NOTL MAX T - T).U(K)
C
C              2) SOIL MOISTURE FILTER MODEL
C                    S(K) = S(K-1) + 1/TC(UT*(K) - S(K-1))
C                    U*(K)  = UT*(K). (S(K)/SMAX)EXP P
C
C                  * * * * * * * *
C
C   READ THE TIME CONSTANT AND EXPONENT FOR THE SOIL MOISTURE
C   FILTER, AND THE NUMBER OF DATA POINTS
C
      DIMENSION T(52),UTST(52),  W(52),U(52),S(52),USTAR(52)
      TC=6.0
      P=1.5
      N=48
C
C   READ THE WEEKLY TEMPERATURE FILE AND THE RAINFALL-
C   STORAGE FILE
C
      DO 50  K=1,N
      READ(5,110) T(K)
 110  FORMAT(F4.0)
      READ(10,100) U(K), W(K)
 100  FORMAT(2F12.6)
  50  CONTINUE
      TMAX =  100.
      C = 1.4
C
C   PROCESS THE RAW RAINFALL THROUGH THE TEMPERATURE
C   MODEL AND WRITE THE FILTERED SERIES IN FILE 13
C
      DO 90 K=1,N
C     UTST(K)=U(K)
C     U(K)=1
      UTST(K)  = C/100.*(TMAX -T(K))*U(K)
 90   WRITE(13,100) UTST(K)
C
C   START THE SOIL MOISTURE FILTERING PROCESS
```

```
C   START THE SOIL MOISTURE FILTERING PROCESS
C   SET THE ANTECEDENT MOISTURE TO ZERO AND
C   DEVELOP THE SOIL MOISTURE SERIES RECURSIVELY
C   ALSO, FIND OUT THE MAX SOIL MOISTURE LEVEL
C
      SMAX=-1000.
      S(K-1)=0.
      DO 20 K=1,N
      S(K) = S(K-1) + 1./TC*(UTST(K) - S(K-1))
      IF(SMAX.LT.S(K)) SMAX=S(K)
   20 CONTINUE
C
C   FILTER THE TEMPERATURE FILTERED RAINFALL
C   THROUGH THE SOIL MOISTURE MODEL
C
      DO 40 K=1,N
   40 USTAR(K) = UTST(K)*(S(K)/SMAX)**P
C
C   WRITE DOWN THE FILTERED INPUT IN FILE 12
C   ALONG WITH THE CORRESPONDING WATER STORAGE
C   LEVELS IN PREPARATION OF USING 'CAPTAIN'
C
      WRITE(12,70)
   70 FORMAT('  45')
      DO 30 K=4,48
      WRITE(12,71) USTAR(K),W(K)
   71 FORMAT(2F12.6)
   30 CONTINUE
      STOP
      END
C
C                    - - - * * * - - -
```

```
C   PROGRAM 2:
C   ---------
C
C     PURPOSE: STOCHASTIC SIMULATION OF THE WATER STORAGE MODEL OF
C              THE DAM AND EVALUATION OF WITHDRAWAL POLICIES
C
C     INPUTS:  PARAMETER ESTIMATES OF THE SYSTEM AND NOISE MODELS;
C              THE VAR-COV MATRICES OF THE ESTIMATES; AND THE
C              RAINFALL SERIES AND THE TEMPERATURE SERIES
C
C     PROCESS: GENERATION OF NORMAL RANDOM VARIATES WITH THE
C              PARAMETER ESTIMATES. PREDICTION OF THE STORAGE,
C              BUILDING IN THE WITHDRAWAL POLICY AND
C              SIMULATION OF EITHER 186 OR 2055 TIMES.
C
C     COMMENT: THE PROGRAM EVALUATES THE IRRIGATION POLICY WHERE
C              THE INITIAL WITHDRAWAL IS MADE ON THE 23RD WEEK.
C              HOWEVER, IT COULD BE EASILY MODIFIED FOR AN
C              ALTERNATIVE POLICY. THE PROGRAM WITH THE WITHDRAWAL
C              DECISION RULES REMOVED COULD BE USED FOR THE
C              MONTE CARLO ANALYSIS OF THE TRANSFER FUNCTION MODEL.
C
C                        * * * * *
C
C   INITIALISE THE COUNTERS FOR THE PROBABILITY DENSITY
C   FUNCTIONS AND THE SUM OF SQUIRES AND VARIANCES FOR
C   THE AMOUNT AND NUMBER OF IRRIGATIONS.
C   READ THE NUMBER OF DATA POINTS AND THE NUMBER OF CLASSES
C   IN THE PROBABILITY DENSITY FUNCTIONS
C
      DIMENSION U(100),S(100),USTAR(100),TSTAR(100),
     * WSTAR(100),PSI(100),Y(100),       T(100),KOUNT(100),
     * K22(100),K48(100),SUMY(100),SEY(100)
C
      MM = 70
      MW = 45
      DO 10 L=1,MM
      KOUNT(L) = 0
      K22(L) = 0
   10 K48(L) = 0
      SUMIR = 0
      SUMSQ = 0
      WADSUM = 0
      WSUMSQ = 0
C
C   READ THE TIME CONSTANT, AND EXPONENT OF THE SOIL
C   MOISTURE FILTER AND THE NUMBER OF DATA POINTS
C   READ THE TEMPERATURE FILE AND THEN THE RAINFALL FILE.
C
      TC=6.0
      P=1.5
      N=48
      DO 50  K=1,N
      SUMY(K) = 0
      SEY(K) = 0
      READ(5,110) T(K)
  110 FORMAT(F4.0)
      READ(10,100) U(K)
  100 FORMAT(F12.6)
   50 CONTINUE
```

```
C   WRITE ON TOP OF EACH OF THE FINAL RESULTS FILE THE NUMBER
C   OF DATA POINTS IT CONTAINS FOR THE PURPOSES OF PLOTTING
C
      WRITE(12,70) MM
 70   FORMAT(I4)
      WRITE(13,70) MW
C
C   PROCESS THE RAW RAINFALL THROUGH THE TEMPERATURE
C   FILTER MODEL AND  OBTAIN THE TSTAR SERIES
C
      TMAX =  100.
      C = 1.4
      DO 90 K=1,N
 90   TSTAR(K)= C/100.*(TMAX -T(K))*U(K)
C
C   CARRY OUT THE SOIL MOISTURE FILTERING PROCESS ON
C   THE TEMPERATURE FILTERED RAINFALL SERIES IN ORDER TO
C   OBTAIN THE ULTIMATE FILTERED RAIN INPUT SERIES, USTAR
C
      SMAX=-1000.
      S(K-1)=0.
      DO 20 K=1,N
      S(K) = S(K-1) + 1./TC*(TSTAR(K)- S(K-1))
      IF(SMAX.LT.S(K)) SMAX=S(K)
 20   CONTINUE
      DO 40 K=1,N
 40   USTAR(K) =TSTAR(K)*(S(K)/SMAX)**P
C
C
C   CARRY OUT A NUMBER OF SIMULATIONS, 2055 TIMES AS
C   DONE HERE
C
      DO 1000 JOB =1,2055
C
      NIR = 0
C
C   GENERATE INDEPENDENT STANDARD NORMAL VARIATES AND COMPUTE
C   STOCHASTIC ERROR TERMS FOR THE PARAMETERS A1, B0 AND C
C   GENERATE THE CORRELATED RANDOM NORMAL VARIATES A, B AND C
C
      RN1 = GRAND(0)
      RN2 = GRAND(0)
      RN3 = GRAND(0)
      E1 = SQRT(7.88E-6)*RN1
      E2 = 4.98E-5/SQRT(7.88E-6)*RN1
     *  + SQRT(4.84E-4 - 4.98E-5**2/7.88E-6)*RN2
      E3 = SQRT(7.28E-3)*RN3
      A = 0.938 + E1
      B = 0.687 + E2
      C = 0.587 + E3
C
C   COMPUTE THE SYSTEM MODEL  STORAGE COMPONENT WSTAR
C   MAKING USE OF THE A AND B VALUES
C
      WSTAR(4) = 8.5
      DO 80 K=5,N
      WSTAR(K) =   A  *WSTAR(K-1) +   B  *USTAR(K)
 80   CONTINUE
```

```
C
C   GENERATE STANDARD NORMAL VARIATE AND COMPUTE THE NOISE
C   MODEL COMPONENT OF THE STORAGE
C
      EK= GRAND(0)*SQRT(3.87)
      PSI(4) =EK
      DO 95  K=5,48
      EK= GRAND(0)
      EK= EK*SQRT(3.87)
      PSI(K) =    C   *PSI(K-1) + EK
   95 CONTINUE
C
C
C    BUILD IN THE DECICION RULES FOR WITHDRAWAL
C    FOR LAND PREPARATION AND PLANTING
C
      SUMW = 0
      DO 30 K=4,N
      IF(K.NE.23) GO TO 1050
      IF(U(22).GE.80.0) GO TO 1050
      XV = 0.8*U(22) + 0.2*U(23)
      RR = 5.0 - XV/10.
      IF(RR.LE.0.0) GO TO 1050
      SUMW = RR
      NIR = 1
 1050 CONTINUE
C
C    BUILD IN THE DECISION RULES FOR WITHDRAWAL
C    DURING IRRIGATION SEASON
C
      IF(K.LT.25) GO TO 1090
      IF(K.GT.36) GO TO 1090
      IF(U(K-1).GE.80.0) GO TO 1090
      IF(U(K-2).GE.80.0) GO TO 1090
      IF(U(K).GE.10.0) GO TO 1090
      IF(U(K).GT.5.0.AND.U(K-1).GT.10.0) GO TO 1090
      IF(U(K).GT.5.0) SUMW = SUMW + 1.0
      NIR = NIR + 1
      IF(U(K).GT.5.0) GO TO 1090
      SUMW = SUMW + 1.5
 1090 CONTINUE
C
C   COMPUTE THE PREDICTED STORAGE
C   COMPUTE THE SUM OF SQUIRES AND VARIANCES OF THE
C   STORAGE LEVEL AT EACH WEEK, NUMBER OF IRRIGATIONS
C   AND THE AMOUNT OF IRRIGATION
      Y(K) =WSTAR(K) + PSI(K)
      Y(K) = Y(K) - SUMW
C     IF(Y(K).LT.0.) Y(K) = 0.
      SUMY(K) = SUMY(K) + Y(K)
      SEY(K) = SEY(K) + Y(K)**2
   30 CONTINUE
C     WRITE(6,73) NIR
C
      WADSUM = WADSUM + SUMW
      WSUMSQ = WSUMSQ + SUMW**2
      SUMIR = SUMIR + FLOAT(NIR)
      SUMSQ = SUMSQ + FLOAT(NIR)**2
C
```

```
C      WRITE(6,72) Y(48)
C
C
C   COMPUTE THE FREQUENCY DISTRIBUTIONS FOR THE OVERALL
C   STORAGE AS WELL AS AT THE 22ND AND 48TH WEEKS
C
      L=0
      DO 120 I = -18,120,2
      XI = I-2
      XIP2 = I
      L = L + 1
      DO 119 K =4,N
 119  IF(Y(K).GE.XI.AND.Y(K).LT.XIP2) KOUNT(L) = KOUNT(L) + 1
      IF(Y(22).GE.XI.AND.Y(22).LT.XIP2)  K22(L) = K22(L) + 1
 120  IF(Y(48).GE.XI.AND.Y(48).LT.XIP2)  K48(L) = K48(L) + 1
 1000 CONTINUE
C
C   COMPUTE THE LOWER BOUND, MEAN AND UPPER BOUND OF STORAGE
C   FOR EACH WEEK AND WRITE IN THAT ORDER IN FILE 13
C
      DO  140  K= 4,48
      SUMY(K) = SUMY(K)/2055.
      SEY(K) = SEY(K)/2055.- SUMY(K)**2
      SEY(K) = SQRT(SEY(K))
         SUMM = SUMY(K) - SEY(K)
         SUMP = SUMY(K) + SEY(K)
 140  WRITE(13,72) SUMP,SUMY(K), SUMM
C
      XK22M = 0
      XK22V = 0
      XK48M = 0
      XK48V = 0
C
C
C   COMPUTE THE PROBABILITY OF OCCURENCE OF STORAGE
C   IN DIFFERENT CLASSES FOR THE 22ND AND 48TH WEEKS.
C   COMPUTE THE MEAN AND VARIANCE OF STORAGE AT
C   THE 22ND AND 48TH WEEKS.
C
      DO 200 I=1,MM
      XKALL=KOUNT(I)/FLOAT(2055*45)
      XK22= K22(I)/FLOAT(2055)
      XK48 = K48(I)/FLOAT(2055)
C
C   WRITE DOWN THE PROBABILITY DENSITY FUNCTIONS OF OVERALL
C   STORAGE AND THOSE OF 22ND AND 48TH WEEKS IN THAT ORDER
C   IN FILE 12.
C
      XI = 2*I - 21
      XK48M = XI*XK48  + XK48M
      XK48V = XI**2*XK48  + XK48V
      XK22M = XI*XK22  + XK22M
      XK22V = XI**2*XK22  + XK22V
 200  WRITE(12,72) XK22,XK48, XKALL
  72  FORMAT(3F12.6)
C
```

```
C    WRITE DOWN THE MEAN AND THE STANDARD DEVIATION OF STORAGE
C    IN THE 22ND AND 48TH WEEKS
C
C      WRITE(6,73) XK48V,XK48M,XK22V,XK22M
  73   FORMAT()
C
       XK48V = SQRT(XK48V - XK48M**2)
       XK22V = SQRT(XK22V - XK22M**2)
C
       WRITE(6,1125) XK22M,XK22V,XK48M,XK48V
 1125  FORMAT(' MEAN AND STD DEV  DISTN AT WK22 ARE',2F12.4,/
      *' MEAN AND STD DEV DISTN AT WK48 ARE',2F12.4)
C
C    COMPUTE AND WRITE THE MEAN AND VARIANCE FOR THE AMOUNT OF
C    IRRIGATION AND FOR THE NUMBER OF IRRIGATIONS
C
       AVNI = SUMIR/2055.
       SENI = SUMSQ/2055. - AVNI**2
C      SENI = SQRT(SENI)
       WRITE(6,1150) AVNI,SENI
 1150  FORMAT(' AVER NUMBER OF IRRIGNS =',F12.4,/
      *' VARIANCE OF NO OF IRRIGNS =',F12.4)
C
       AVAMI = WADSUM/2055.
       SEAMI = WSUMSQ/2055. - AVAMI**2
C      SEAMI = SQRT(SEAMI)
       WRITE(6,1151) AVAMI,SEAMI
 1151  FORMAT(' AVER AMOUNT OF IRRIGN =',F12.4,/
      *' VARIANCE OF AMNT OF IRRN =',F12.4)
       STOP
       END
C
C                        - - -  * * *  - - -
```

```
C
C   PROGRAM 3:
C   ---------
C
C     PURPOSE: TO AGGREGATE THE DAILY RAINFALL RECORDINGS ON A WEEKLY
C              BASIS AND ALSO TO WRITE DOWN THE APPROPRIATE WEEKS'
C              RAINFALLS FOR THE MONTE CARLO ANALYSIS
C
C     INPUTS:  TWO FILES CONTAINING DAILY RAINFALL RECORDS, THE FIRST
C              OF WHICH IS IN INCHES HAVING 559 LINES OF DATA, EACH
C              CONTAINING 14 OBSERVATIONS. THE SECOND FILE HAS THE
C              SAME FORMAT BUT IN MM. THE FIRST LINE OF DATA FOR
C              EACH YEAR IS THAT OF THE FIRST FORTNIGHT OF SEPTEMBER
C
C     PROCESS: THE RAINFALL OF THE FIRST FILE IS CONVERTED INTO MM.
C              AFTER PROCESSING, THE APPROPRIATE 48 WEEKS' RAINFALLS
C              (FROM 4TH WEEK OF SEPT - 3RD WEEK OF AUGUST) ARE RECORDED.
C              THE TOTAL FOR THE FIRST 20 WEEKS' AND THE ANNUAL RAINFALL
C              FOR EACH YEAR ARE ALSO COMPUTED.
C
C     COMMENT: DEVELOPED TO PROCESS 20 YEARS' RAINFALL RECORDS. HOWEVER,
C              IT IS OPEN TO HANDLE ANY AMOUNT OF DATA.
C
      C                    * * * * *
C
      DIMENSION DATA(14), WKD(2)
      XMULT= 25.4
      KNT = 0
    5 READ(5,10,END=100) DATA
   10 FORMAT(14F5.2)
      WKD(1) = 0.
      WKD(2) = 0.
      KNT = KNT + 1
      KONT = MOD(KNT,26)
      IF(KONT.EQ.1) SUMR = 0
      IF(KONT.EQ.1) SUM20 =0
      DO 20 I =1,7
      WKD(1) = WKD(1) + DATA(I)*XMULT
   20 WKD(2) = WKD(2) + DATA(I+7)*XMULT
      SUMR = SUMR + WKD(1) + WKD(2)
      SUM20 = SUM20 + WKD(1) + WKD(2)
      IF(KONT.NE.1.AND.KONT.NE.2.AND.KONT.NE.0)
     * WRITE(10,30) WKD
      IF(KONT.EQ.2) WRITE(10,30) WKD(2)
      IF(KONT.EQ.0) WRITE(10,30) WKD(1)
   30 FORMAT(F12.6)
      IF(KNT.GE.459) XMULT=1
      IF(KONT.EQ.10) WRITE(6,30) SUM20
      IF(KONT.EQ.0) WRITE(6,30) SUMR
      GO TO 5
  100 CONTINUE
      STOP
      END
C
  C                    - - - * - - -
```

```
C   PROGRAM 4:
C   ---------
C
C     PURPOSE: PLOTTING OF THE WATER STORAGE MODEL AND THE
C              PROBABILITY DENSITY FUNCTIONS OF THE STORAGE
C
C     INPUTS:  TWO FILES , IN UNITS 12 AND 13. THE PROBABILITY
C              DENSITY SERIES OF THE 22ND WEEK,48TH WEEK AND
C              THE OVERALL STORAGE IS FOUND IN FILE 12 WITH THE
C              NUMBER OF DATA POINTS ON TOP, WHILE FILE 13 HAS THE
C              UPPER BOUND, MEAN AND LOWER BOUND STORAGE SERIES
C              ALSO WITH THE NUMBER OF POINTS INDICATED ON TOP.
C
C     COMMENT: THE PROGRAM MAKES USE OF THE 'ANUPLOT' ROUTINE
C              TO PLOT WITH THE CALCOMP 960 PLOTTER.
C
C                          * * * * *
C
C   ASK FOR THE FILE UNIT TO BE PLOTTED WHETHER 12 OR 13.
C
      DIMENSION  A(52), X(52), Y(52), Z(52)
      M = 30
      WRITE(6,11)
   11 FORMAT(' ENTER UNIT NO TO READ DATA')
      READ(5,22) IUNIT
   22 FORMAT( )
      READ(IUNIT,3) N
      IF(IUNIT.EQ.13)  M = N
    3 FORMAT(I4)
      DO 4  I =1, N
      A(I) = I*2
      IF(IUNIT.EQ.13)  A(I) = I
    4 READ(IUNIT,5) X(I), Y(I), Z(I)
    5 FORMAT(3F12.6)
C
C
      CALL PLOTC(15,31,65.,0)
      CALL FACTOR(1.5)
      CALL PLOT(1.,1.,-3)
      IF(IUNIT.EQ.12) CALL SCALE(A,6.0,M,1)
      IF(IUNIT.EQ.13) CALL SCALE(A,5.,N,1)
      CALL SCALE(X,5.0,M,1)
      CALL SCALE(Y,5.0,M,1)
      CALL SCALE(Z,5.0,N,1)
      IF(IUNIT.EQ.12) CALL AXIS(0.,0.,'STORAGE LEVEL',-13,
     *6.0,0.,A(M+1),A(M+2))
C
      IF(IUNIT.EQ.13) CALL AXIS(0.,0.,'WEEKS',-5,5.0,0.,
     *A(N+1),A(N+2))
C  CALCULATE MAXIMUM OF X(N+2), Y(N+2), Z(N+2)
C
      XMAX = X(N+2)
      IF(Y(N+2).GT.XMAX) XMAX = Y(N+2)
      IF(Z(N+2).GT.XMAX) XMAX = Z(N+2)
C
```

```
C
      IF(IUNIT.EQ.13) X(N+2) = XMAX
      IF(IUNIT.EQ.13) Y(N+2) =XMAX
      IF(IUNIT.EQ.13) Z(N+2) = XMAX
      IF(IUNIT.EQ.13) X(N+1) = 0
      IF(IUNIT.EQ.13) Y(N+1) = 0
      IF(IUNIT.EQ.13) Z(N+1) = 0
      IF(IUNIT.EQ.12) CALL AXIS(0.,0.,'PROB DENSITY',12,
     *5.0,90.,X(M+1),X(M+2))
      IF(IUNIT.EQ.13) CALL AXIS(0.,0.,'MEAN STORAGE',12,
     *5.0,90.,X(N+1),XMAX)
C
      IF(IUNIT.EQ.12) CALL LINE(A,X,M,1,0,3)
      IF(IUNIT.EQ.13) CALL DASHL(A,X,M,1)
      IF(IUNIT.EQ.12) CALL PLOT(15.,0.,-3)
C     IF(IUNIT.EQ.12) CALL CURSOR(XX,YY,IAA)
      IF(IUNIT.EQ.12) CALL AXIS(0.,0.,'STORAGE LEVEL',-13,
     *6.,0.,A(M+1),A(M+2))
      IF(IUNIT.EQ.12) CALL AXIS(0.,0.,'PROB DENSITY', 12,
     *5.0,90.,Y(M+1),Y(M+2))
      CALL LINE(A,Y,M,1,0,3)
      IF(IUNIT.EQ.13) GO TO 66
      DO 65 I=1,N
 65   A(I)=I*2
 66   CONTINUE
C
      IF(IUNIT.EQ.12)  CALL SCALE(A, 5.0,N,1)
      IF(IUNIT.EQ.12) CALL PLOT(15.,0.,-3)
C     IF(IUNIT.EQ.12) CALL CURSOR(XX,YY,IAA)
      IF(IUNIT.EQ.12) Z(N+2)=.03
      IF(IUNIT.EQ.12) CALL AXIS(0.,0.,'STORAGE LEVEL',-13,
     *5.,0.,A(N+1),A(N+2))
      IF(IUNIT.EQ.12) CALL AXIS(0.,0.,'PROB DENSITY',12,
     *5.0,90.,Z(N+1),Z(N+2))
      IF(IUNIT.EQ.12) CALL LINE(A,Z,N,1,0,3)
      IF(IUNIT.EQ.13) CALL DASHL(A,Z,M,1)
      CALL PLOT(0.,0.,999)
      STOP
      END
C
C                            - - -  *  - - -
```